A3525
NO.
170

United States Department of Agriculture

Forest Service

North Central Forest Experiment Station

General Technical Report **NC-170**

The Fractal Forest: Fractal Geometry and Applications in Forest Science

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1992 Folwell Avenue
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Manuscript approved for publication July 20, 1994
1994

ACKNOWLEDGMENTS

Egolfs V. Bakuzis, Professor Emeritus at the University of Minnesota, College of Natural Resources, collected most of the information upon which this review is based. We express our sincere appreciation for his investment of time and energy in collecting these articles and books, in organizing the diverse material collected, and in sacrificing his personal research time to have weekly meetings with one of us (N.L.) to discuss the relevance and importance of each referenced paper and many not included here. Besides his interdisciplinary approach to the scientific literature, his extensive knowledge of forest ecosystems and his early interest in nonlinear dynamics have helped us greatly. We express appreciation to Kevin Nimerfro for generating Diagrams 1, 3, 4, 5, and the cover using the programming package *Mathematica*. Craig Loehle and Boris Zeide provided review comments that significantly improved the paper.

Funded by cooperative agreement #23-91-21, USDA Forest Service, North Central Forest Experiment Station, St. Paul, Minnesota.

YES. I HAVE A THREE-PART QUESTION, EACH PART OF WHICH HAS THREE PARTS, EACH PART OF EACH PART OF WHICH HAS THREE MORE PARTS...
MICHAEL O.

Q. & A. TIME AT THE FRACTAL GEOMETRY CONFERENCE

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CONTENTS

1. Introduction 1

2. Description of Fractals 2
- Fractal Objects 2
- Fractal Processes 3
- Iteration 4
- Initiator-Generator 4
- Classical Fractals 4
- Julia and Mandelbrot Sets; the Complex Plane 6
- Natural Fractals 6

3. Fractional Dimension 7
- Dimension in General 7
- The Fractal Dimension 7
- The Hausdorff and Other Dimensions 9
- Computing the Fractal Dimension 9

4. Fractals and Scale 12
- Scaling in Nature 12
- Other Scaling Techniques and Their Relation to Fractals 13
- Scale-free Fractals 14

5. Fractals and Self-similarity 15
- The Parts and the Whole 15
- Local vs. Global 16

6. Fractals and Complexity 16
- Jaggedness and Boundary 16
- Complex Pattern and Self-organization 17
- Noise 17

7. Applications to Forest Systems 18
- Leaf Shape 18
- Tree Branching 19
- Tree Crowns 20
- Tree Models 20
- Vegetation Patches 22
- Size and Density of Organisms Related to Habitat 22
- Species Diversity 23
- Animal Movement 24
- Forest Disturbance 26
- Modeling Soils with Sierpinski Carpets 27
- Spatial Heterogeneity and Sampling 28
- D and Shoreline Erosion 30
- Data Storage and Compression 31

8. Fractal Applications in Managing Forest Landscapes 32

Bibliography 34

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1. INTRODUCTION

Fractal geometry is a new branch of mathematics with roots in set theory, topology, and the theory of measures. As a young science (Bunge 1968), the theory of fractals is more like a collection of examples, linked by a common point of view, than an organized theory. But the mathematical underpinnings are developing; and researchers in other fields are applying the concepts and techniques, thereby extending fractal geometry beyond its mathematical beginnings (Mandelbrot 1983).

Despite its newness, fractal geometry expands and sparks investigation in physics, chemistry, and biology because it offers something very important—tools for describing and analyzing irregularity. Kinds of irregularity can be classified as new regularities, seemingly random but with precise internal organization. With fractals we see new patterns in the world (Stewart 1988b).

The fractal world view is different from the world view exemplified by calculus, by Leibniz' "principle of continuity" (Mandelbrot 1983). Our tools for a forest that is smooth, continuous, linearly ordered, and at equilibrium have been Euclidean geometry, calculus, the Fourier transform, and the Gaussian curve. But most of what we measure in the forest—crowns, roots, soils, stands, plant-animal interactions, landscapes—is discontinuous, jagged, and fragmented; and our tools, designed for smoothness, have trouble measuring them. Fractal geometry is a tool for discontinuity, a tool for the fractal forest.

We are interested in mechanism as well as measurements. When we ask: What is the mechanism?, we traditionally mean: What is the linear interaction of two or more elements that *cause* something to happen? The mechanisms in fractal systems, however, are probably not linear cause-and-effect; they are complex scaling interactions (West 1987).

Like relativity and quantum theory, fractal geometry brings the observer into the story. Consider the dimension of a ball of string. From far away, the ball looks like a dot (dimension = 0). Closer, it is a solid (= 3). Very close it is a twisted thread (= 1). Then closer yet it is a column (= 3). Closer still it is fine twisted hairs (= 1). Dimension in this example is a function of the observer's location (Briggs and Peat 1989). In sections 2 and 3, we will show how dimension is a function of the observer's choice of measuring unit.

Euclidean geometry is one of the conventions in mathematics and science that is chosen for its utility (Giedymin 1991). But that utility is limited, when Euclidean geometry is faced with the complexity generated by nonlinear dynamical systems (Bohr and Cvitanovic 1987). Fractal geometry is an alternative, a complementary, geometry. But as a new geometry is proclaimed, we need not feel uneasy about venturing beyond Euclid. Mathematician (and geometer) Poincaré (1952) observed, "One geometry cannot be more

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true than another; it can only be more convenient." Different geometries serve different purposes.

This review is a collection of fractal thought and its reference to many natural disciplines. We have used illustrations from work in forestry whenever possible, but ideas about fractals come from many disciplines (e.g., physiology, chemistry, physics). So, while a leaf can illustrate fractal dimension, a ferromagnet is a good example of self-similarity. The purpose of our interdisciplinary approach is to make this wide-ranging material accessible to researchers in forestry. This is not a fractal cookbook, although references to specific techniques are provided. Because of the fast-breaking nature of this new field, we focus on helping readers quickly understand concepts they can apply to their own research. As a result, we have left out the full theoretical discussion these concepts deserve.

We followed the semantic convention that if an object or process has fractal properties, it is called a fractal. Properties (Bunge 1979) can be component properties ("a tree has fractal shape") or relational properties ("stand boundaries have fractal shape"). Although there is fair agreement in the literature that the fractal properties are fractional dimension, scale independence, self-similarity, and complexity, the necessary and sufficient conditions for an object or a process to have fractal properties have yet to be determined.

Our paper begins with a general description of fractals in section 2, followed by more detailed exposition of four fractal properties in sections 3-6: fractal dimension (with a section on how to compute the fractal dimension), scale, self-similarity, and complexity.

Section 7 describes specific applications to forest systems. In assembling this section, we used a wide definition of forestry research. Applications for fractals in forestry include forest ecology, mensuration, wildlife habitat, microclimatology, vegetation mapping, soil science, and geology. For example, a tree can be modeled as a fractal, predicting total wood volume, stem surface area, and number of branch tips. Damage patterns in forests and arrangements of tree crowns are fractals (Loehle 1983).

In section 8, we offer thoughts on the research potential for fractals in forest resource management. Where needed in our review, more detailed information is presented inside a box near the general discussion. These boxes can be skipped in an overview reading.

Although the danger exists that exciting new fields may raise false hopes (Mayer-Kress 1986), the evidence for the potential of fractals is overwhelming (Zeide 1990). Fractals may be the most promising direction for forest research in its task of representing the forest (Zeide 1990).

2. DESCRIPTION OF FRACTALS

Fractal Objects

Self-similar objects that cannot be described in common Euclidean fashion, that are non-uniform in space, are fractals. Self-similarity means that as magnification (the scale) changes, the shape (the geometry) of the fractal does not change (Gefen 1987). Besides self-similarity, fractals have infinite detail, infinite length, fractional dimension, and they are generated by iteration (Briggs and Peat 1989).

Mandelbrot (1967), who called fractals to the world's attention, had difficulty in defining them. In 1983 he approached the definition with the aid of measure theory, using the Hausdorff-Besicovich measure of dimension. Rigorous definitions of topological and Hausdorff dimensions, with which fractals are usually described, are very difficult to understand (Cipra 1989); there are mathematical complications. The simplest way to define fractals is via the self-similarity assumption. Despite many attempts, the definition of "fractal" is still not definitive.

The continuing development of fractal definition within measure theory does not hamper the utility of the concept. The usefulness of the fractal concept stems from its ability to describe apparently random structures within a precise geometry (Orbach 1987). Mandelbrot (1983) offers a way to characterize fractals by the fractal dimension, D. Fractal objects cannot be adequately described by two dimensions or three dimensions or n dimensions of whole numbers because fractal dimensions are fractions.

Mandelbrot's famous example of a fractal is a coastline's rich twists and turns; a coastline is not a simple one-dimensional line, nor is it two dimensional like a plane. A coastline, Mandelbrot suggested, has fractional dimension, also called fractal dimension, meaning that it has a dimension somewhere between a line's one dimension and a plane's two dimensions. The computed fractal dimension of coastlines ranges from 1.15 to 1.25.

The mathematical concept of fractal dimension follows; fractional dimension is more thoroughly described in section 3. A Koch curve is the simplest way to show how the fractal dimension is derived (diagram 1). Start with a straight line divided into three equal segments. Replace the center segment with two segments that are each the same length as the original center segment. Now repeat the process with each of the four new lines. That is, divide each line into three segments and replace the middle segment with two segments. Continue for an infinite number of times. The Koch curve is self-similar: any portion magnified by a factor of three looks like the previous iteration. At each step the length scale changes by a factor of $b = 3$, called the scaling factor. In the diagram below, the number of lines changes from 1 to 4 to 16 to 64. The number of basic units, then, changes by a factor of $N = 4$. The fractal dimensionality is:

$$N = b^D \text{ , or } D = \ln N / \ln b.$$

For the Koch curve, the fractal dimensionality is:

$$D = \ln 4 / \ln 3 \quad 1.26$$

(Gefen 1987).

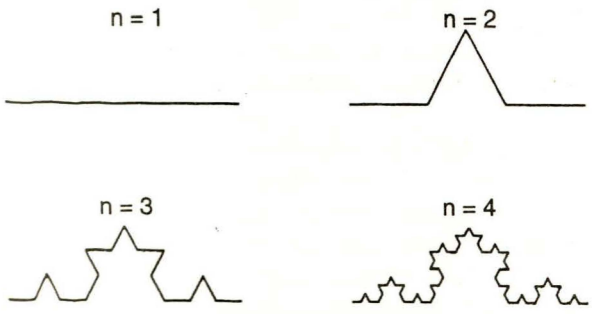

Diagram 1.—*The generation of a Koch curve. Divide each line segment into three segments, and replace the center segment with two segments the same length as the segment being replaced. Repeat.*

Because of fractal dimensionality, the length of a Koch curve and other fractal curves like the coastline varies by the unit of measurement (Richardson 1960). As the unit of measurement decreases, the length of the line goes to infinity (Mandelbrot 1983).

To use an example from the forest, the perimeter of a maple leaf is not smooth; it is jagged. With a video imaging system, Vlcek and Cheung (1986) generated a one-pixel-thick computer image of a leaf. They measured the leaf outline with different measurement lengths; the measurement lengths each had different numbers of pixels. The values for the leaf's perimeter differed depending on which measurement length did the measuring.

Just as the length of a coastline depends on the resolution of scale, on the size of the measuring unit, so does the perimeter of a leaf depend on the size of the measuring unit—in the above case, on the number of pixels that mark off the perimeter of the leaf.

Fractal geometry can also be applied to planes and even to amorphous solids. An amorphous solid has less than three dimensions because of the openness of the structure. There are "holes" in the solid. These kinds of structures are often heterogeneous, or irregular, as well (Orbach 1987).

Measuring a fractal object (e.g., the perimeter of a leaf) depends on the scale of the measuring unit. As the scale becomes infinitely fine, the value of the attribute diverges toward infinity. Because measurement of fractal objects depends on scale, the usual statistical concepts (e.g., sample mean) need to be re-thought when applied to the measurement of fractal objects (Loehle 1983).

Fractal Processes

Fractals are geometrical objects or constructs, but they may also have dynamical properties. That is, processes may have fractal properties. Diffusion processes (Orbach 1987), like diffusion-controlled reactions with geometrical constraints (Kopelman 1988), and vibration distributions (Orbach 1987) have fractal dimensionality. In the frequency distribution of vibrations, for example, the fractal dimension describes the relationship between vibrational

frequency and the spatial extent of vibration. A structure can have Euclidean mass properties and yet have fractal dynamic properties. This happens if the structure is "filled in" with atoms, but not all the atoms are participating in the vibrational activity (Orbach 1987).

A fractal process is one that cannot be characterized by a single time scale, just like a fractal structure does not have a single scale length. Fractal processes have many component frequencies, producing a broad spectrum of frequencies. So a fractal time series can be investigated with spectral analysis, a technique that unravels the complexity of frequencies. These techniques are applied to many physiological processes (West and Goldberger 1987).

Heterogeneity, self-similarity, and absence of a characteristic length of scale are therefore three properties applicable to dynamics as well as to structures. And besides geometric fractals and temporal fractals, there is the category of statistical fractals. For the latter, the statistical properties of the whole are the same as that of its parts (West 1987).

Iteration

Iteration, repeating a process over and over, is a key idea in the whole field of dynamic systems and fractals (Devaney 1988). The Koch curve above was generated by the repetition of a single step, replacing the center segment with two segments.

Tree branching is an example of iteration in forestry. The growth of a modular organism like a tree is determined by rules of iteration and the reaction of each growing point to the local conditions around it (Franco 1986).

A modular organism has repeated units of structure at varying levels of organization (Halle *et al.* 1978, Harper *et al.* 1986). Fractals are also self-similar sets composed of modules or pieces similar to the entire set on lesser scales (Harrison 1989).

Computers iterate easily and are thereby useful in the generation and study of fractals (Harrison 1989). Because of iteration, complicated forms do not have to be generated by complicated processes. Fractals are complex (they contain much detail) yet simple (they can be simply generated) (Briggs and Peat 1989).

Initiator-Generator

A fractal may be generated by performing a repeated operation on a beginning structure. The beginning structure is called the initiator. The repeated change is called the generator.

Demonstrating the concepts, diagram 2 shows some formal examples of initiators—the beginning shapes—in the first column. The generators are the changes made to each line segment in the initiator; the generators are sketched above the arrows (Arlinghaus 1985).

On the first row of the diagram, the application of the generator to the initiator gives three copies of the initiator at reduced scale, in the second column. When the generator is applied a second time, the next figure (third column) has three images of the previous figure. Continuing on, the (n + 1)st figure contains three copies of the nth figure. The fractal dimension measures shape with shifting scale; it measures self-similarity (Arlinghaus 1985).

Classical Fractals

There are other types of "classical" fractals, such as the Koch curve and Julia sets (see box on page 6) that are heavily used in mathematical studies and in the development of theory about fractals. They are useful for illustrating fractal characteristics before moving on to more complicated forms. In forestry we usually wish to untangle fractals, not generate them. That is, we want to find the initiator and the generator. The question becomes: What is the algorithm that produces this pattern? By first learning how fractals are generated, we may better understand how to untangle them.

A Cantor set is constructed by removing the middle third of a line segment, then removing the middle third of those segments, and continuing to remove middle thirds (Stewart 1989b). Cantor dust, formed by the remaining points, has a fractal dimension between the zero dimension of a point and the one dimension of a line (Mandelbrot 1983).

A Peano curve (diagram 3) is a line that fills up a plane (Kramer 1970), and has dimension 2.

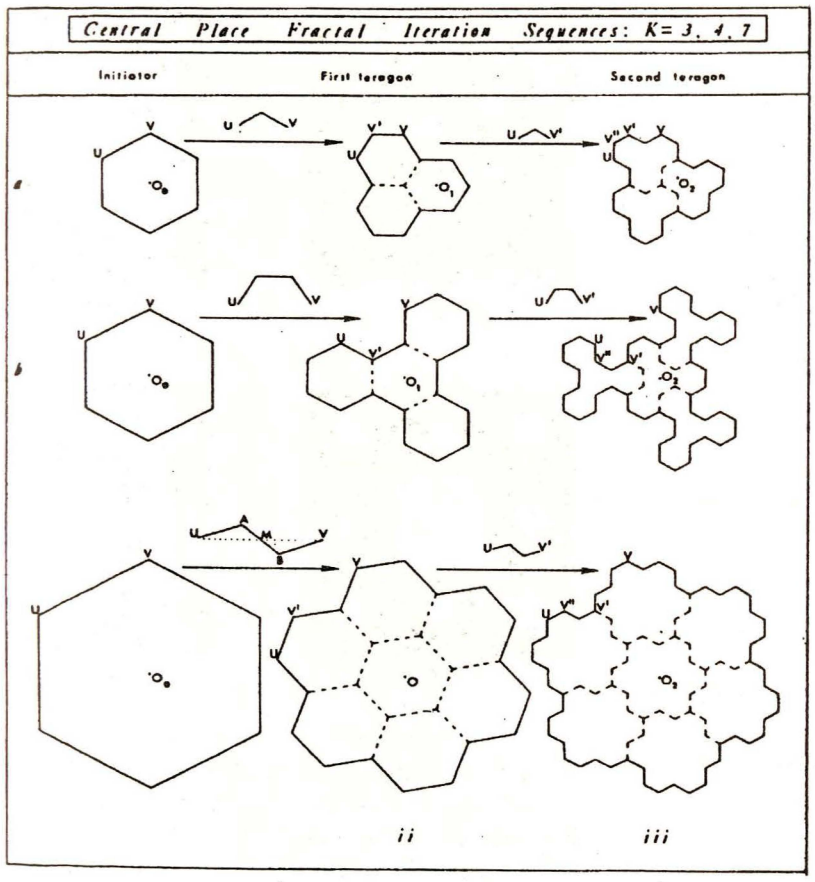

Diagram 2.—*Initiators and generators. Each line segment in the beginning structure, the initiator, is replaced by a new form, the generator. In the second step, the line segments in the new structure are replaced in the same way, and a third structure emerges. (Reprinted with permission from: Lach, S. 1985. Fractals take a central place. Geografiska Annaler. 67B: 83-88.)*

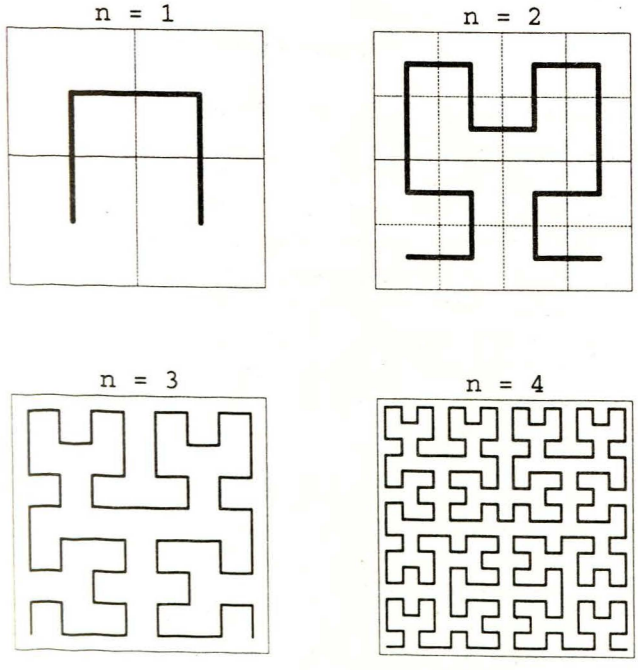

Diagram 3.—*Generation of a Peano curve. The rule, "move the line to the center of each of four squares," is followed as each square is successively divided into four squares. Eventually the line fills the whole plane, and thereby has dimension two.*

Sierpinski carpets are generated by a recursion in which successively smaller holes are cut from the plane to give a self-similar pattern at levels smaller than the initial plane (diagram 4). Tyler and Wheatcraft (1990a) modeled soil types with Sierpinski carpets. The dark holes are the pore spaces among soil grains. The fractal dimensions of these carpets can be used to estimate the area of pores greater than a given size. The holes in the face of a Sierpinski carpet reduce its dimensionality to less than a plane, between dimensions 1 and 2.

Julia and Mandelbrot Sets; the Complex Plane

The examples of fractal generation in section 2 were generated by iteration of lines or figures. Equations can also be iterated to generate fractals, and Julia sets and the Mandelbrot set are classic examples of the results of iterating equations.

Strang (1986) explains how Julia and Mandelbrot sets are generated. Start with a relation

$$f(z) = z^2 + c,$$

where both z and c are complex numbers. Iterate, starting from $z_0 = x_0 + i\,y_0$, and compute $z_1 = z_0^2 + c$, $z_2 = z_1^2 + c$, etc. The sequence, z_0, z_1, z_2, etc. has, according to Strang, three possibilities:

z_n can approach a limit; it can approach infinity; or it can do neither. Different Julia sets are generated by changing the value of z_0 when iterating the above relation. A Julia set is the set of points in the z_0 plane for which z_n stays bounded as n approaches infinity (Strang 1986).

The Mandelbrot set is related to the Julia set in the following way. The relation

$$f(z) = z^2 + c,$$

is iterated as described above, but, c is varied rather than z_0. Each value of c leads to a Julia set in the z_0 plane, but the new set M is in the c-plane and is an "index" to all Julia sets. Strang observes that the Mandelbrot set ..."may be the most complicated figure ever to be studied, and partly understood, by humans." However, its definition, he says, remains remarkably simple: "M contains all numbers c for which the sequence $z_0 = 0$, $z_1 = c$, $z_{n+1} = z_n + c$, ... stays bounded" (Strang 1986).

The rich variation and form in the Mandelbrot set and Julia sets come about through nonlinearity within rigorous organization (Douady 1986). They are excellent illustrations of a common theme in the study of fractals: very simple, nonlinear processes generate very complex structures (diagram 5).

Programs are available for generating Mandelbrot sets, Julia sets, and other fractals on personal computers (Strang 1986, Peitgen and Saupe 1988).

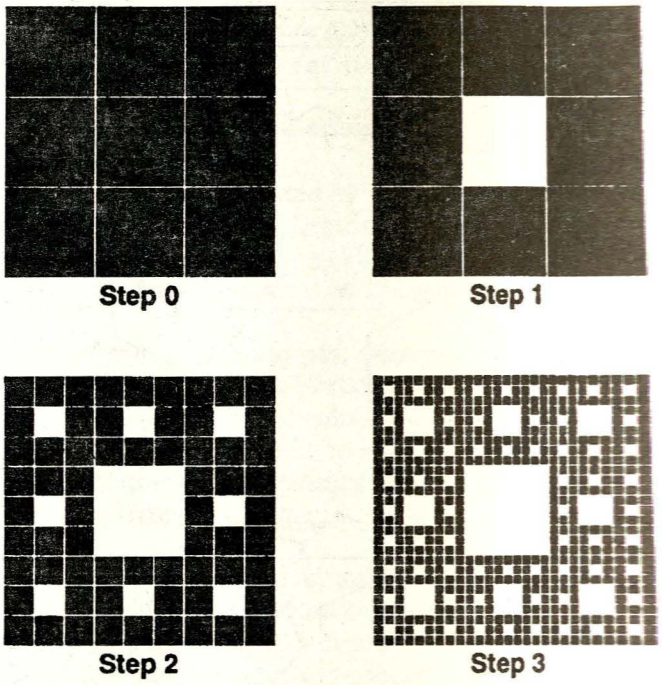

Diagram 4.—*Generation of a Sierpinski carpet. Remove the center square and then remove the center square from the remaining squares. Repeat. The dimension of these figures falls between one for a line and two for a plane.*

Natural Fractals

Nature adds the random element to fractals. Deterministic fractals are the mathematical fractals discussed above. Mathematical fractals can be generated with a computer, with a mapping or a rule repeated over and over. Natural fractals normally have some element of randomness. Fractals generated with a computer program that includes random elements simulate natural phenomena more closely than fractals generated by deterministic methods (Saupe 1988).

Fractals not exactly self-similar, not exactly the same at all scales, are called multifractals (Stewart 1989b). The dimensional description of multifractals requires two or more dimensional exponents. Examples of multifractals are complex surfaces and interfaces, and fluid flow in porous media (Stanley and Meakin 1988). Forest fractals, fractals in nature, are most likely multifractals, not strictly self-similar at every scale.

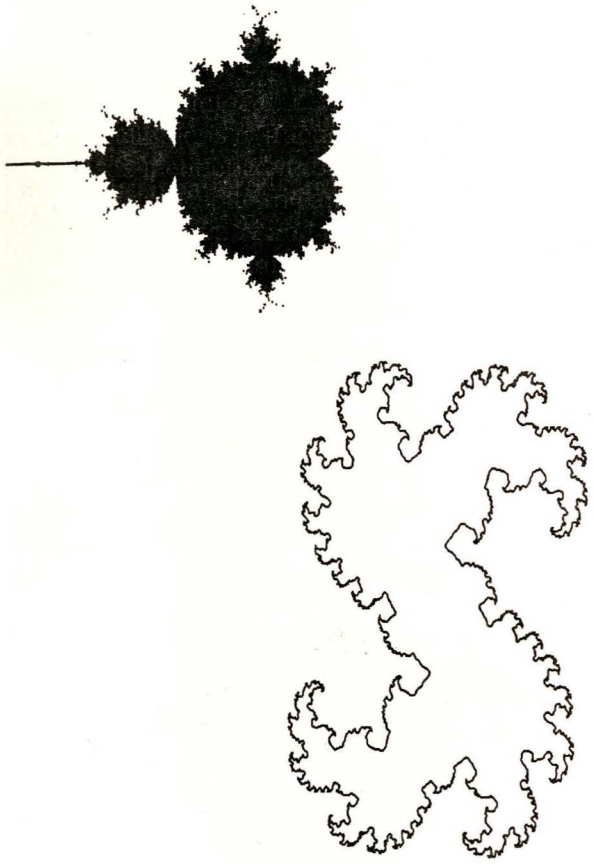

Diagram 5.—*The Mandelbrot set (upper figure) and a Julia set (lower figure). The figures are generated by the iteration of simple, nonlinear equations.*

In the next four sections, we describe four important properties of fractals—fractional dimension, scale independence, self-similarity, and complexity.

3. FRACTIONAL DIMENSION

Dimension in General

Geometry makes explicit our intuitions about space. Classical Euclidean geometry is a first approximation of the structure of physical objects. Fractal geometry extends classical geometry with a new language (Barnsley 1989). Dimension is a strong center in the language of all geometries, yet dimension is elusive and very complex (Mandelbrot 1967). The loose notion of dimension turns out to have many mathematical ramifications (Mandelbrot 1983) and definitions (Mendes-France 1990). Irregularity and fragmentation cannot properly be understood by defining dimension simply as a number of coordinates.

The Fractal Dimension

The difficulties of defining dimension in general carry over to definitions of fractional dimension (Cipra 1991), as mentioned in the previous section. However, despite the ambiguity about the underlying philosophy and mathematics, fractal dimensions are numbers that precisely specify deviation from the expected scaling rule (Pagels 1985). They are measures of the relative degree of complexity (Briggs and Peat 1989).

It is helpful to compare Euclidean dimensions and fractal dimensions (also see box on page 9). When the number of measurement units is graphed against the size of measurement unit, the dimension is read from the slope. Typical Euclidean objects have slopes of 1 (length), 2 (area), and 3 (volume) (diagram 6). Natural objects fall between these slopes; their dimensions are fractions (Zeide 1990).

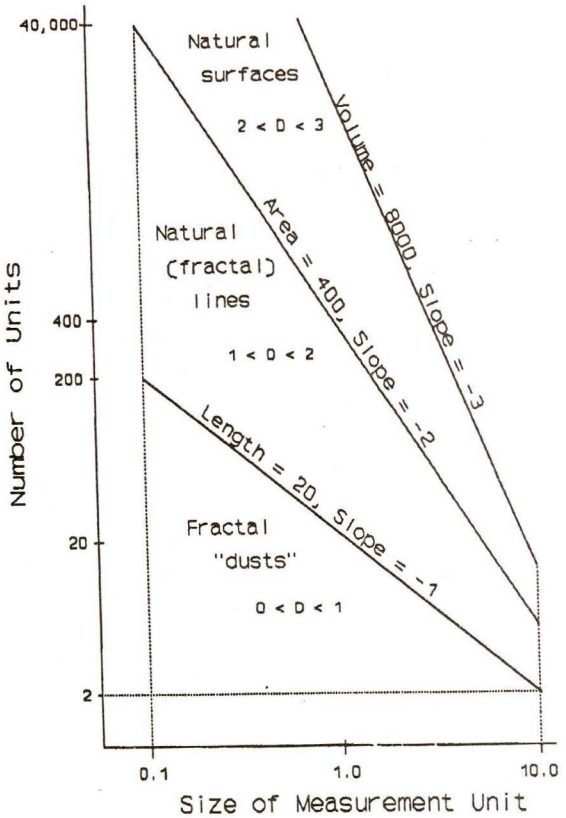

Diagram 6.—*Dimension is the relationship between the size of measurement unit and number of measurement units. Euclidean dimensions are whole numbers. Fractal dimensions are fractions (Zeide, Boris. 1990. Fractal geometry and forest measurements. Gen. Tech. Rep. PNW-GTR-263. Portland, OR: U.S. Department of Agriculture, Forest Service, Pacific Northwest Research Station: 260-266.)*

In another comparison, Tsonis and Tsonis (1987) looked at three kinds of dimensionality: Euclidean, topological, and fractal (diagram 7). Topology is a branch of mathematics that treats form as flexible and compressible. In topology, a wiggly line is identical to a circle; both have dimension 1. By definition, topology cannot discriminate between crooked lines and straight lines (Mandelbrot 1983). For the four objects in the diagram, a straight line, a crooked line in a plane, a crooked line in a volume, and a crooked plane in a volume, dimensions in the three different geometries are identical for only one object: the straight line.

The non-integral dimension may be a relative notion. A tangled worm in a stagnant pond has a trajectory with a dimension near 2. The same worm in a current would have a stretched trajectory, and its dimension would approach 1. Dimension can depend on the object's state of motion, on the frame of the observer's reference, and on the observer's yardstick (Cherbit 1990).

The observer chooses the unit of measurement, and the length of the object will depend on the unit chosen. But this does not mean that measurement is arbitrary. Correspondence between unit and length is maintained. Length becomes a process rather than an event, a process controlled by a constant parameter unique for each object measured. This parameter is called fractal dimension (Zeide 1990).

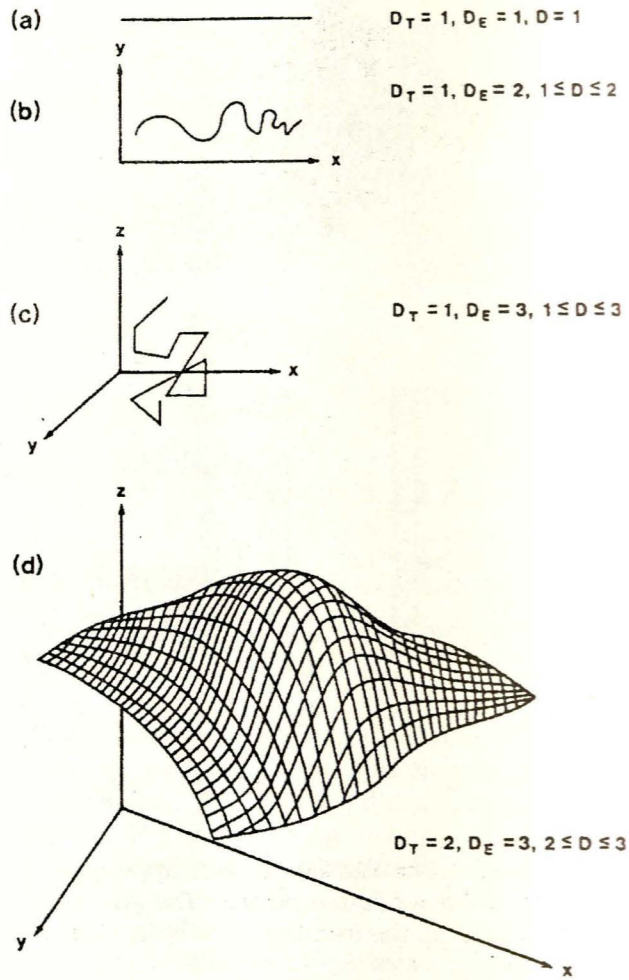

Diagram 7.—*Euclidean (D_E), topological (D_T), and fractal (D) dimensions of lines and surfaces. The dimension of an object can vary depending on the geometrical system in which it is measured. (Reprinted with permission from: Tsonis, A.; Tsonis, P.A. 1987. Fractals: a new look at biological shape and patterning. Perspectives in Biology and Medicine. 30: 355-361.)*

The Hausdorff and Other Dimensions

By Mandelbrot's definition, a fractal set is a set with its topological dimension less than its Hausdorff dimension. The Hausdorff dimension, in the realm of geometric measure theory, can be compared to Euclidean dimensions, except that the Hausdorff dimension has been defined with more mathematical rigor (Harrison 1989).

D, the fractal dimension or the Hausdorff-Besicovitch dimension, is

$$D = \log N / \log r$$

where N is the number of measurement steps and r is the scale ratio (or self-similarity ratio). A curve of D = 1 approaches D = 2 when it becomes so complex that it effectively takes up the whole plane (Phillips 1985).

Several kinds of fractal dimensions have been identified by mathematicians. So far, this kind of rigor in definition has not been necessary in fractal applications to biological problems.

Sometimes scaling properties emerge only asymptotically, that is, at the extremes, at the microscopic level or at the upper limit. The dimensions calculated for these asymptotic regions have been called the inner (microscopic) and outer (upper limit) dimensions, or the local and global dimensions. The Hausdorff dimension and the capacity dimension are inner fractal dimensions (Naudts 1988).

The *capacity* dimension is defined by box-counting. As the size of the box decreases, the number of boxes increases by the fractal exponent. The capacity measure depends on metric properties of the space, because the length measure is crucial. This dimension is often called the Hausdorff, but there is a technical difference (Rasband 1990).

The *information* dimension depends on the natural probability measure of the set. For unequal probabilities, the information dimension will be less than or equal to the capacity dimension (Rasband 1990).

For the *correlation* dimension, the number of pairwise correlations between points in a set scales as the size of the set is raised to some power. As set size decreases, the number of correlations will decrease too. This dimension takes the density into account. The correlation dimension is less than or equal to the information dimension, which is less than or equal to the capacity dimension. For many fractals, the three exponents will be equal (Rasband 1990).

Computing the Fractal Dimension

Loehle (1990) warned that finding natural fractal dimensions can be difficult and computationally intensive. Very complicated fractals are difficult to measure, but computing the fractal dimension of a crooked line, for example, is relatively simple.

In fractal theory, length measurements for a crooked line vary with the size of the measurement unit. Therefore, the line must be measured with different sizes of measurement units. Over a range of scales, the length of the line will differ, depending on the unit of measurement. The relationship between the length of the line and the unit of measurement is predictable. That relationship is the fractal dimension.

In more formal terms, begin an estimate of the length of a crooked line by stepping dividers with step size s_1 along the line. Count the number n_1 of steps required to traverse the line, giving a length estimate of $n_1 s_1$. Repeat the process with a smaller step size, s_2, for another length estimate, $n_2 s_2$, which may be greater than or equal to the previous one. The more jiggles in the line, the greater the increase in length between the two estimates. Then:

$$D = \log(n_2/n_1) \; / \; \log(s_1/s_2).$$

If the line is smooth, then halving the step size or sampling interval will require precisely twice as many steps, and D will equal 1 (the Euclidean dimension of a line); but if the line is crooked, then D will be greater than 1 by some fraction (Goodchild and Mark 1987). The more crooked the line, the higher the fractal dimension. As mentioned in the previous section, the very complex Peano curve completely fills the plane and has dimension 2.

When the unit of measurement is plotted against length for several scales on log-log paper, the slope is the key. If the slope is a straight line, the fractal dimension is the same over a wide range of sampling intervals (Goodchild and Mark 1987).

Perimeters and outlines are simply closed crooked lines, and there are many examples of interest in the forest (e.g., Loehle 1983). Calculating the fractal dimension becomes automated when the object is photographed by a video

camera and the perimeter length is counted by computer program (Goodchild and Mark 1987). Vlcek and Cheung (1986) calculated the fractal dimension of six leaves from eight trees representing seven species. The slope of the plotted data gives the fractal dimension (diagram 8).

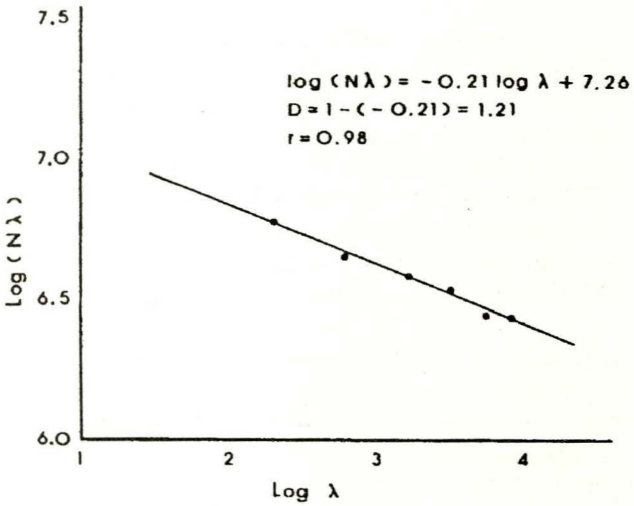

Diagram 8.—*Calculating the fractal dimension (D=1.21) of a silver maple leaf from the slope of the log pixel length versus the log number of lengths. (Reprinted with permission from: Vlcek, J.; Cheung, E. 1986. Fractal analysis of leaf shapes. Canadian Journal of Forest Research. 16: 124-127.)*

For more complicated objects like whole plants, using a grid system may be more efficient than moving different-sized measurement units around the outline. A photograph of the object can be placed under grids of three sizes (diagram 9). The logarithm of the number of squares touched by the figure is plotted against the logarithm of the number of squares on one side of the grid. The slope of the line is the fractal dimension (May 1988). Morse *et al.* (1985) explain this technique in more detail.

Fractal dimensions of areas and surfaces can be obtained using pixels or line segments. Square pixels of different sizes will measure the area of an irregular patch. A rough surface will have a fractal dimension greater than 2 (Goodchild and Mark 1987). Surfaces may be complicated, but rough surfaces can be sectioned before the fractal dimension is computed. To obtain the fractal dimension of a rough surface, make serial sections perpendicular to the surface and measure the length of the section boundary

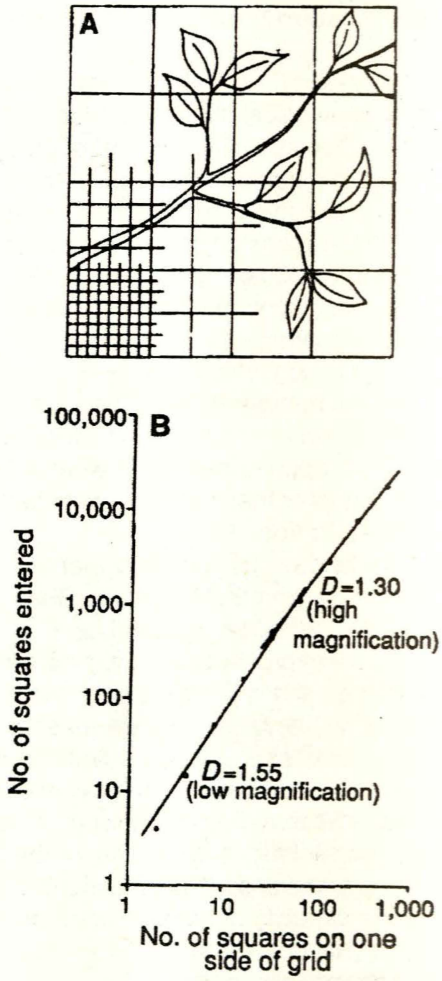

Diagram 9.—*Using a grid to compute the fractal dimension of a plant. With three or more grid sizes, count the number of squares that cover the object (A). Plot grid size against number of squares entered. Read the slope (B). (Reprinted with permission from: May, R. 1988. How many species are there on Earth? Science. 241: 1441-1449.)*

using line segments. Another approach is to make sections parallel to the surface and measure the length of perimeters of "island" and "lakes" (Cahn 1989).

Box counting is a method used to describe dispersed objects. In two dimensions, a grid is placed on the map of dispersed objects, and the number of penetrated boxes is counted. The box counting is repeated for different scales, thereby providing data for computing the fractal dimension (Schroeder 1991). Frontier (1987) describes a method of using circles to compute the fractal dimension of dispersed objects (diagram 10).

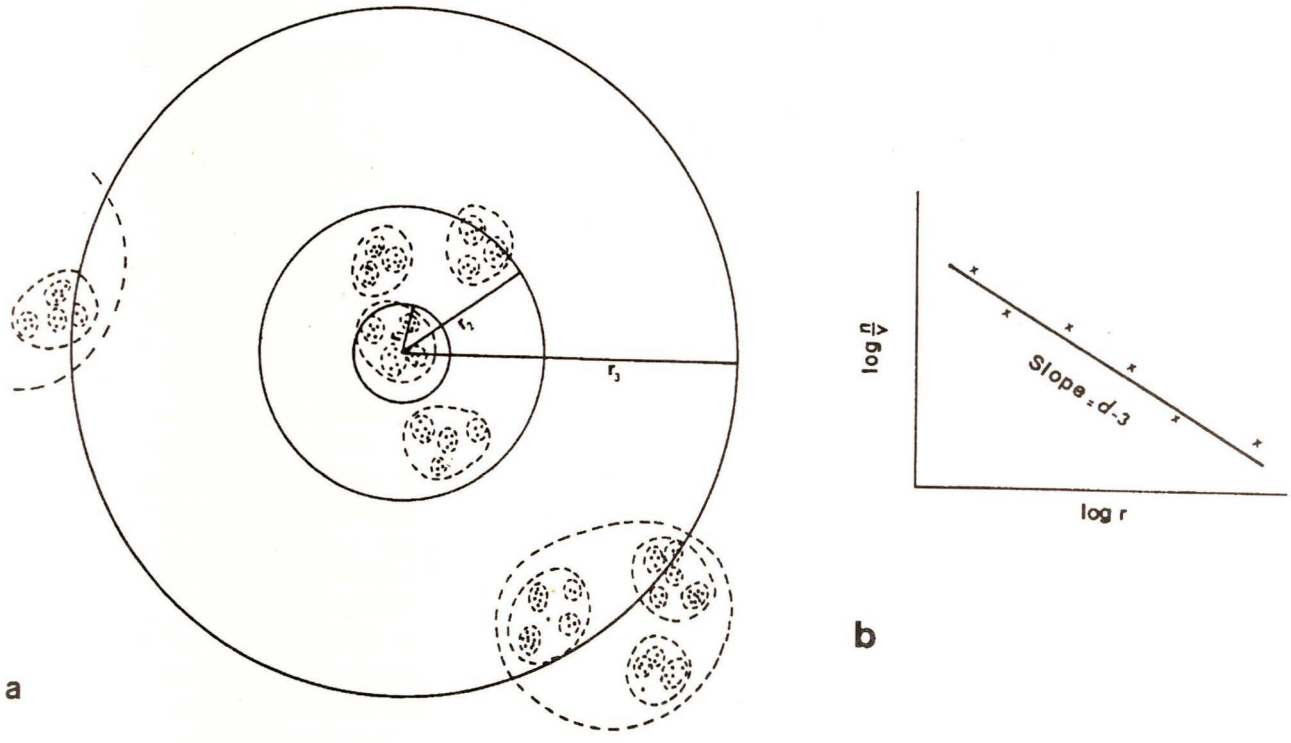

Diagram 10.—*Measuring the fractal dimension of the patchy distribution of biomass through space with spheres of varying radii, r. If d is the fractal dimension, the number of points intercepted by the spheres is n α r. The density of points inside the spheres is n/V α r^{d-3}. The slope is read from the log-log plot. (Reprinted with permission from: Frontier, S. 1987. Applications of fractal theory to ecology. In: Legendre, P.; Legendre, L., eds. Developments in numerical ecology. New York, NY: Springer-Verlag: 335-378.)*

Loehle (1990) uses a box-counting technique to describe animal home range. For each point where an animal is observed, a circle centered on this point is drawn to represent the area in which the animal searches for prey during a short time interval. Next, a grid is laid over the map; the height of each grid square equals the number of circles that overlap the square. Algorithms are provided for computing the fractal dimension of the resulting 3-dimensional surface.

Wavelet analysis is a new technique replacing Fourier analysis for complex time series data. The Fourier transform has been used for situations of irregularity and fragmentation; but it is scale dependent, and no criteria have been established for choosing scale (Loehle 1990). Wavelet analysis is a mathematical technique used for interpreting seismic and accoustic data. In contrast to the Fourier transform, wavelet analysis separates position and scale as independent variables and allows identification of self-similar properties of fractal objects (Argoul *et al.* 1989).

Some measures for computing fractal dimension are summarized in table 1.

Table 1.—*Measurements for computing the fractal dimension*

Object/Process	Example	Measurements	Citation
Crooked line	Coastline	Unit lengths	Mandelbrot 1967
Outline	Leaf	Automated unit length (pixel lengths)	Vlcek and Cheung 1986
Outline	Plant	Grid sizes	Morse *et al.* 1985
Surface	Landscape	Pixel areas	Goodchild and Mark 1987
Dispersed material	Algae	Circles	Frontier 1987
Movement	Home range	Box	Loehle 1990
Solids	Crystals	Box	Fowler *et al.* 1989
Time series	Population size	Wavelet	Argoul *et al.* 1989

4. FRACTALS AND SCALE

Fractal geometry is the first mathematical theory that explicitly uses the concept of observation scale. Observation scale is the gap in the dividers by which we measure distance and/or time, like pixel length in measuring leaf perimeters, and sphere size in measuring dispersed biomass. The gap is normally the prerogative of the investigator. In building a fractal object, a process is repeated from scale to scale, following a "cascade." However, the reality of the scale is hidden by the process itself because without a reference the scale is unknown. In a fractal, all scales are equivalent and undiscernible from the form itself (Frontier 1987).

Scaling in Nature

The structure of the world has scales of many different sizes. Although some events differing greatly in size do not influence each other, some phenomena do depend on events at many scales (Wilson 1979).

An example from the forest illustrates how scale separates research efforts. Plant physiologists work with individual leaves; ecologists work with individual plants; foresters and meteorologists look at vast areas of vegetation. Because of scale differences, these scientists may disagree about mechanisms of function, like transpiration for example. Conflict of interpretation arises from conflict of scale. Results of one group do not apply to the problems studied by another group. The information is not interchangeable (Jarvis and McNaughton 1986). Even on one small site, herbs and trees can hardly be studied together because of differences in scale (Hoekstra *et al.* 1991).

Before the fractal approach, many scales were a barrier for physicists who wished to understand and describe turbulence. Now turbulence is modeled as a multifractal, reflecting a complex cascading structure. Statistical models were insufficient because they did not give topological information about the location of the intermittent structures in space (Argoul *et al.* 1989).

For mathematicians and physicists, scaling is a primary focus. Ecologists, who deal with intuitively familiar phenomena, are likely to choose scales appropriate to person-centered perception (Wiens 1989, Hoekstra *et al.* 1991).

Temporal and spatial scales are sometimes vague when ecologists and foresters discuss patterns of vegetational change. For example, how large an area and how long a time are needed for succession? Large- and small-scale areas may or may not be affected by large- and small-scale disturbances (Nakamura 1989).

Once spatial and temporal scales have been identified, what is the relation between the two? Both are required in models designed for predictability. Where are the standard functions that define the appropriate units for such space-time comparisons in ecology? Continuous linear scales may not be appropriate for discontinuous processes (Wiens 1989).

Combining time and space scales is a challenge for many scientists. For example, soil-litter dynamics change in space and time. Over centuries, there is slow accumulation with change due to succession. Over years, there are seasonal processes. Over days, there are rapid fluctuations due to winds and arthropods (Nakamura 1989).

Growth rates may increase and decrease over time. When an organism develops in stages, there may be scale changes at specific points (Zhirmunsky and Kuzmin 1988).

Scales may contract or expand from one hierarchical level to another (Zhirmunsky and Kuzmin 1988). Although some correlation exists between scale and level of organization (May 1989), there are no *a priori* meaningful levels of scale (Cherbit 1990).

Hierarchy theory emphasizes linkages among scales. We need standardized ways to define and detect scales. Scaling issues may be particularly important in the study of interactions among species. Arbitrary scales (e.g., quadrat size) may limit the detection of interaction (Wiens 1989).

Predictions of spatially broad-scale systems cannot be made with temporally small-scale samples because the time-scale grain is too fine to catch the major movements in the dynamics of the system. This is a severe problem in resource management, where large-scale policies flow from short-term studies (Wiens 1989).

Scaling is a common problem in science. Several scaling techniques have been developed that should be compared and contrasted with fractal methods (see box).

Other Scaling Techniques and Their Relation to Fractals

Similitude, dimensional analysis, allometry, and renormalization are scientific techniques developed to deal with scale. They resemble fractal analysis in some ways and differ from it in others.

The technique of similitude analysis aims for equations with the least number of variables by combining variables and making them dimensionless (Hillel and Elrick 1990). After a solution is found to a reduced formula, the solution can be applied to many other systems that may be physically different, but are "scale models" of each other (Miller 1980). For example, in soils, similitude defines scale-invariant relationships for water in porous media. A characteristic length scale emerges that reflects the sizes of solid particles and the sizes of pores in a particular geometric arrangement. Similitude uses this length scale to produce scaled transport coefficients and water potentials in porous media (Sposito and Jury 1990).

The Fibonacci series has been used for scaling in the lung, but these kinds of classical similitude scaling techniques assume an underlying uniformity of the process or structure, a uniformity that is usually missing (West and Goldberger 1987). Nor can similitude analysis be used for systems in which some of the variables are unknown (Snyder 1990).

In dimensional analysis, interconversion of units is the goal. That is, ratios between parameters are substituted for actual values (including the units) of those parameters (Gunther and Morgado 1987). All variables and parameters are written down and then separated into dimensionless groups. Dimensional analysis is different from similitude analysis because it directly reduces the appropriate differential equations themselves, along with the attached boundary conditions of a problem. An infinity of possible physical systems is "collapsed" into one reduced solution that describes them all (Miller 1980).

Power laws and allometry are techniques for describing the relationship between variables as the scale changes. Allometry is a power law specifically related to biological growth. Power laws are self-similar.

$$f(x) = cx^a$$

where c and a are constants. If x is re-scaled (multiplied by a constant), f(x) is still proportional to x^a. Power laws lack natural scale. They are called "scale free" or "true on all scales," or "happening on all scales" (Schroeder 1991).

In Huxley's original equation

$$Y = aW^b$$

Y is any biological function, a is an empirical parameter, W is body weight, and b is an exponent (Gunther and Morgado 1987).

Allometric development is a relationship between any two properties of a process in which the relationship is defined by a power function. In allometric relationships there is usually a problem deciding which of two variables is the dependent and which is the independent variable (Zhirmunsky and Kuzmin 1988).

Fractal analysis may resemble allometry because both approaches use power functions, but the two are different. Allometry's power function relates linear rates of growth between body parts. But for fractals the variables are size and number of units of measurement (Zeide and Pfeifer 1991).

Some allometric relationships link organism size with other parameters, like metabolic rates (Sernetz *et al.* 1985). Physiologists have long expressed metabolic rate as a function of body size with the allometric power law

$P = aM^b$

with P the rate of energy transformation, M the body mass, and $b = 3/4$. But there is disagreement about the value of b (Calder 1987).

To use a growth model from forestry, according to the self-thinning "rule," growth and mortality over time in a crowded, even-aged stand will have a slope of -3/2 on a double-logarithmic graph. Attempts to model the -3/2 rule geometrically have had to assume that plants maintain the same average shape, that is, grow isometrically. But average plant shape is not independent of mass across the plant kingdom. Comparisons across the plant kingdom (shrubs vs. herbs, for example) are hampered by scale problems (Weller 1989).

Renormalization is a technique for preserving information while changing scale. If something is renormalizable, it is self-similar. Renormalization recognizes self-similarity at scale changes and thereby ties directly into fractal theory. Renormalizability may be a fundamental property of nature, like the principle of special relativity is a fundamental property of nature (Pagels 1985).

If a process or object is self-similar, then a part will look like the whole. Scale disappears. Magnifications of smaller and smaller parts will stabilize, that is, they will look almost identical. Renormalization functions like a microscope that zooms in on self-similar structure (Stewart 1989b).

Renormalization divides a problem with multiple scales into smaller problems, each on a single length of scale. Three steps are repeated many times. First, the system is divided into many equal sections. Then the variable is averaged for each section, and that one average replaces all the other variables in the section. After this, the spacing in the new system is larger than the old system—the variables are farther apart. For the third step, the original spacing is restored by reducing back to the dimensions of the original system (Wilson 1979).

Renormalization blurs out the smaller features of the system but preserves the larger ones. At the same time, it provides information about the behavior of distinct but related systems in which the fundamental scale of length gets larger with each iteration. After the first transformation, the fluctuations at the smallest scale have been eliminated, but those slightly larger, with a scale of roughly three times the original spacing, can be seen more clearly. The resulting system reflects only the long-range properties of the original system, with all finer scale fluctuations eliminated (Wilson 1979).

Scale-free Fractals

In a fractal situation, "part of a large scale element is equal to the whole of the small scale element" (Nakamura 1989); and this feature of fractals makes them useful in dealing with problems of scale in research.

Domains of scale may be regions of scale change over which the pattern for the phenomenon of interest does not change. Outside the domain, the pattern changes or at least changes linearly with change in scale. Domains are separated by sharp transitions, similar to phase transitions in physical systems. Variability increases with the approach of transition as has been shown for increasing quadrat size, continuous linear transects, and point samples (Wiens 1989).

Fractal geometry may be useful for identifying domains of scale. For fractals, pattern may differ in detail at different scales, but the pattern remains self-similar. The fractal dimension, D, "indexes the scale-dependency of the pattern." The fractal dimension is constant if the processes at fine scale are responsible for the processes at larger scale. A change in fractal dimension signals a change in process. Therefore, regions of fractal self-similarity may represent scaling domains. The goal is to define the algorithms that will translate across scales (Frontier 1987, Wiens 1989).

Fractal dimensions of the physical environment may also change by scale. For example, turbulence is bounded between the scale of the planet and the scale of the molecule. But in between are viscosity, lapping, waves, local currents, and large-scale currents. Organisms adapt to these different scales, resulting in different forms, behaviors, or fractal dimensions (Frontier 1987).

How fractals bridge (quantify) the scale differences between small scale and large scale can be seen in the following example of small insects on large vegetation. Fractal dimensions have been calculated for various vegetational habitats, and they range from 1.3 to 1.8, with an average of about 1.5. For insects that eat the edges of leaves (a one-dimensional approach to the vegetation), a tenfold decrease in the size of the insect is like a threefold increase in the apparently available vegetation. (If the fractal dimension is 1.5, a tenfold reduction in the measurement scale increases the perceived

length by $10^{0.5} = 3$). If insects eat surfaces instead of edges (a two-dimensional approach), the effect is squared: a tenfold decrease in the insect's size is like a tenfold increase in apparent habitat (May 1988).

Fractals allow us to quantify process change over scales (De Cola 1991). The multiplicity of scales can sometimes be decoded by time series analyses, such as in electrocardiograms. The richness of structure shows the "underlying fractal networks" in physiology (West and Shlesinger 1990).

The various time scales or frequencies of a process can be arrayed in a spectrum. For comparison, a simple harmonic oscillator has a spectrum of one frequency, and a random time series has a broad spectrum. But a fractal process or fractal time series is very different, having no characteristic scale. Its frequencies make up an inverse power law spectrum, $1/f$, where f is a frequency (West and Shlesinger 1990).

Frequencies in space correspond to numbers of cycles per meter, rather than cycles per second. For both space and time frequencies, the inverse power law spectrum shows the fractal or scale-invariant nature of the underlying process. Many natural phenomena exhibit this spectrum and thereby this scale invariance. They are called $1/f$ phenomena (West and Shlesinger 1990).

In population biology, the average of population sizes over a few years (an "equilibrium") may really be a stage of cycling over a long time period. That cycle may be moving on an even larger, less frequent cycle (Pimm and Redfearn 1988). Controversies about whether ecological systems are equilibrium or non-equilibrium systems probably can be resolved by the appropriate choice of time scale and the application of fractal techniques (Wiens 1989).

5. FRACTALS AND SELF-SIMILARITY

The Parts and the Whole

The mathematical fractals introduced in section 2 are self-identical—the figures remain the same as the scale changes. The whole of the figure can be reproduced by magnifying some part of it. Fractals in nature are not self-identical, but they are self-similar: the parts resemble the whole.

Although self-similarity is an old idea dating from Leibniz' study of the line in the early 1700's, and winding from Richardson's study of turbulence through Kolmogorov's study of mechanics, self-similarity has come into its own with the investigation of fractals.

It is lack of scale that leads to self-similarity (Schroeder 1991). Self-similarity means that a part of a line, for example, can represent, statistically speaking, the whole line. It also means that two lines with the same fractal dimension will be like each other and perhaps have the same generating mechanism (Vlcek and Cheung 1986). Self-affine means self-similar, but with more than one scaling factor (Schroeder 1991).

Stewart (1989a) described self-similarity more formally in the language of set theory. "A set is self-similar if there exists some finite system of transformations such that the images of the set under these transformations, when combined, yield the original set. In other words, the set can be split into a finite number of pieces, each of which has the same structure as the whole, but on a smaller scale."

The work of D'Arcy Thompson was based on the scaling concept of similitude, on the idea that an underlying process is continuous. But the concept of self-similarity does not require underlying continuity (West 1987).

A real biological object is not infinitely self-similar, but is only self-similar over some range of scale. The lung branches 24 times, and gill branches 4 times. A real tree branches for eight binary steps. Beyond that, the twigs bear leaves with another fractal structure. And at the other end of the scale, an individual tree belongs to a forest, another fractal (Frontier 1987).

A tree is self-similar, with limits. It cannot grow indefinitely or sap could not be supplied to all the leaves. It cannot branch indefinitely because air circulation is required. Foliage tissue is organized as a sponge, a fractal, for most efficient air and sap contact. The number of branching steps optimizes the transfer of matter and energy (Frontier 1987).

Biological fractals are also truncated because morphogenesis is expensive in energy and information. If the structure's function (for example, a sufficient contact surface) is achieved after a few generative steps, then self-similarity is curtailed; trees and lungs have structure with limited steps (Frontier 1987).

Another example of the physical limitations of self-similarity is a particle moving through soil, tracing a fractal line. The length of the actual path depends on the unit of measurement. By zooming in on the path, a finite length becomes visible. This finite length has to exist or the particle would travel an infinite length, a physical impossibility (Tyler and Wheatcraft 1990b).

Fluctuating phenomena in nature are often statistically self-similar structures. With statistical self-similarity, there is equal probability of two events happening. Strike out every other event, and the statistics of the first set will look like the statistics of the second set. If the probabilities are not equal, then adjacent samples are correlated and self-similarity is missing. Where adjacent events are correlated, the smaller sample will have a smaller correlation (Schroeder 1991).

Renormalization theory in physics is closely related to self-similarity. The physics of phase transitions is like the mathematics of Julia sets. This may reflect a basic principle of phase transitions, which are or may be fractal structures (Peitgen and Richter 1986).

Local vs. Global

Because a fractal structure exhibits some degree of self-similarity—the structure is the same at several or many scales—fractal structure has bearing on the relationship between the local scale and the global scale.

Fractals provide a measure of short-range vs. long-range variation. Low D means "domination by long-range controls." Landforms have a range of $D = 1.1$ to 1.3, indicating the influence of large-scale factors like climate and geology. High D values indicate short-range factors like soil chemistry (Phillips 1985).

Systems undergoing phase transition are highly self-similar. Phase transitions, whether from solid to liquid or from magnetic to non-magnetic states, all involve formation or destruction of long-range order (Peitgen and Richter 1986).

Complex fractals produced by the repetition of a simple geometric transformation can turn out completely different by some slight change in the formula (Briggs and Peat 1989). Diffusion through a solid is an example of local changes making global results. When Rosso *et al.* (1990) simulated the diffusion of a substance through a solid, the "diffusion front" had a fractal dimension of 1.75. When studied in detail, the dynamics of the diffusion front revealed macroscopic results (undiffused areas become diffused) from microscopic changes (single particle movements) over very short time periods. Long time periods of little activity were suddenly punctuated by remarkable events. The accumulation of numbers of points at the diffusion front obeyed a power law characteristic of fractional Brownian motion (Rosso *et al.* 1990).

The number of points was sampled at various times. When the sampling time was greater than $a^2/4D$ (a is the inter-site distance and D is the coefficient of diffusion), mean fluctuations of numbers of points stabilized at the frequency characteristic for fractal phenomena, $1/f$ (Rosso *et al.* 1990).

6. FRACTALS AND COMPLEXITY

Jaggedness and Boundary

Many distribution patterns in time and space are not Gaussian, with well-defined means and variances. Rather, they are likely to be "jagged on every scale" (May 1989). Discontinuity, heterogeneity, and complexity characterize many biological systems, and fractal geometry is a way to approach complex systems (West and Goldberger 1987). The major problem of irregularity in spatial data, relating what is happening here to what is happening there, may be addressed using notions of fractal structure (De Cola 1991). Dimension is a measure of complexity (Mendes-France 1990), but there is no value of D that marks the point on the continuum between complex and simple systems (Phillips 1985).

Biological systems may require the complex boundaries provided by fractals. Contact zones between interacting parts of an ecosystem may be enhanced by fractal geometry. Energy,

matter, and information may flow more easily from one segment to another (Frontier 1987).

Webs of organisms connect through exchange of matter and energy. Energy flow couples with transport mechanisms between points of different energy levels. A fractal wall configuration and transport system accelerates the energy flow and the cycling of matter through the system. For trees, for example, the fractal canopy forms the contact between the atmosphere and the chlorophyll web; the fractal root system forms the contact between the tree and nutrients in the soil. The lungs and gills may be other examples of fractal structures that exchange energy and matter between the organism and its environment (Frontier 1987).

The littoral zone is an important example of a fractal, complex ecosystem boundary. Pond shape and lake shape are related to biological properties like productivity. Lake shape has been described in terms of a ratio between shoreline length and water volume, but it is actually shoreline fractal dimension that should be correlated with other properties. The littoral zone brings together the primary producers and the decomposers, accelerating the cycling of matter. This contact zone between two ecosystems increases as the fractal dimension of the shoreline increases (Frontier 1987).

Life may be defined by interaction and the flow of matter, energy, and information through interfaces, interfaces that are dimensionally constrained, interfaces that are fractals. Fractal organization is especially visible in ecology, the science of interaction between populations and environment (Frontier 1987).

Complex Pattern and Self-organization

How does pattern form from a disordered environment? Is pattern formation in nature the result of unique causes and effects governing each phenomenon, or do unifying principles underly the formation of pattern (Ben-Jacob and Garik 1990)?

Pattern may form from the interaction of microdynamics with macrodynamics. Some unifying principles are emerging from growth patterns. An example of the self-organization of complex patterns are systems with a diffusion field like the one discussed in section 5. Patterns group into typical morphologies, regardless of the system or scale (Ben-Jacob and Garik 1990).

Biomass is often distributed in hierarchical patches, and patterns like these can be studied using the fractal approach. Frontier (1987) suggests sampling plankton along a ship trajectory. The fractal object, the plankton in three dimensions (the swarm), intersects with the linear sample. Because the intersection of two fractal objects is a fractal, the dimension of the intersection is equal to the sum of the dimensions of the two original fractals, minus the dimension of the space in which the two fractals interact. The same rule applies to the intersection of a fractal with a non-fractal object. If D and d are the fractal dimensions of, respectively, the swarm and the linear sample, $d = D + 1 - 3 = D - 2$, then $D = d + 2$. The geometry of plankton patches is a function of water turbulence (Frontier 1987).

For large-scale spatial heterogeneity, the landscape is the appropriate spatial unit. Here is where the physical and biological interact, where population interacts with population and with the abiotic to form pattern. For landscape ecology, spatial pattern is the driving force determining system function. Pattern is studied at large scale to relate it to ecological processes studied at smaller scale. Parameters have to be developed that define spatial pattern in a way that hooks up with the underlying biophysical process (Krummel 1986).

Noise

Randomness, noise, is a component of fractal formation in nature. Noise contributes to the complexity of fractals. Mandelbrot (1983) highlighted this in his coastline example. The events leading to the formation of a coastline are impossible to reconstruct. Geomorphological processes happen "through many ill-explored intermediates." Chance enters, not only on the microscopic level, like in the Brownian molecular motion of molecules, but also on the macroscopic level (Mandelbrot 1983).

Noise is the uncertainty in complex systems. The probability distribution function describes complexity, independently of the underlying

mechanisms. As normally distributed or lognormally distributed systems increase in complexity, they come to resemble $1/f$ systems (diagram 11). The area under the curve of lognormal distributions increases as complexity increases, until it reaches a $1/f$ distribution, a straight line with negative slope. The $1/f$ and the lognormal distributions will overlap more and more as complexity increases. When many mechanisms are working at many scales and the main process is accomplished by many subprocesses, the overlap of the lognormal and $1/f$ distributions will be large and the distributions will be indistinguishable. This is why the inverse power law can be so universal (West and Shlesinger 1990).

The behavior of $1/f$ phenomena is due to complexity rather than content (West and Shlesinger 1989). The power spectra of many physical variables are "$1/f$-like," with the form f^{-l} $(0.5 < l < 1.5)$ over many degrees of frequency. These include loudness fluctuations in music and speech, and melody in music (Voss and Clarke 1975).

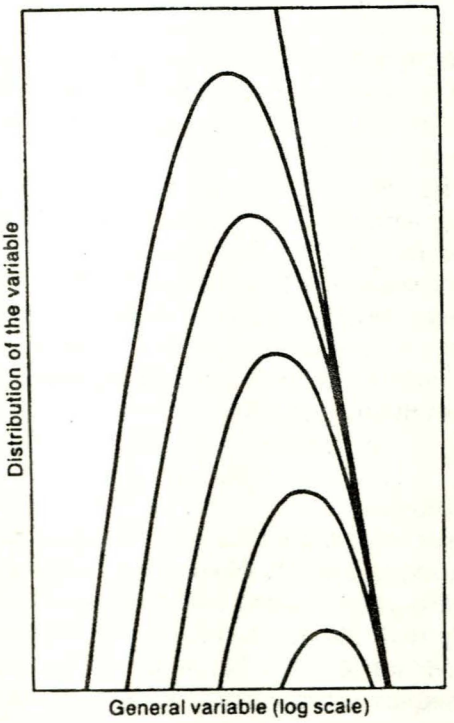

Diagram 11.—*Variability of lognormal and $1/f$ systems. As simple systems with little variability become more complex, the area under the curve increases until the distribution is described by a straight line with negative slope: a $1/f$ system. (Reprinted with permission from: West, B.; Shlesinger, M. 1990. Noise in natural phenomena. American Scientist. 78: 40-45.)*

The power spectrum is related to the autocorrelation function. The $1/f$ power spectrum cannot be characterized by a single correlation time. There is a sloped relationship over time scales (Voss and Clarke 1975).

The spectrum of noise distributed as $1/f$ is called fractal noise because it is scale-free.

7. APPLICATIONS TO FOREST SYSTEMS

As with most young fields, the applications of fractal geometry to the biological sciences have been primarily descriptive. Given fractal geometry's predilection for re-analysis of shape and size, it is not surprising that initial applications to problems in forestry should emphasize shape and size.

Several researchers in forestry-related disciplines have used the concept of fractional dimension (table 2). Fractal dimensions have been computed for a diverse array of things, from tree rings to taxonomic lists, and will be discussed in the sections that follow.

Besides the fractal dimension, there has been some use of fractals and scale, and of fractals and self-similarity. However, many of the studies do not deal strictly with forestry. They belong more formally to the realms of ecology (e.g., species distribution) or general biology (e.g., taxonomy). Nevertheless, the techniques and approaches may be useful to the forest scientist, whose aim is to describe and understand the forest's complexity.

Leaf Shape

Leaf shapes traditionally have been described by linear and/or areal measures, but these measures do not represent natural shapes very well. Vlcek and Cheung (1986) calculated the fractal dimensions of the perimeters of leaves (diagram 12). The nine species they measured were significantly different. More complex leaves had higher D and greater standard deviation among leaves.

Table 2.—*Fractal dimensions from the forest*

Feature measured	Dimension	Reference
Tree ring indices	1.22-1.45	Burrough 1985
Tree ring widths	1.77	Taylor 1988
Oak branching	0.07-1.44	Crawford and Young 1990
Red oak leaf	1.19	Vlcek and Cheung 1986
White oak leaf	1.22	Vlcek and Cheung 1986
Sugar maple leaf	1.18	Vlcek and Cheung 1986
Silver maple leaf	1.21	Vlcek and Cheung 1986
Gingko leaf	1.11, 1.05	Vlcek and Cheung 1986
English elm leaf	1.05	Vlcek and Cheung 1986
American elm leaf	1.04	Vlcek and Cheung 1986
American basswood leaf	1.02	Vlcek and Cheung 1986
White oak leaf	1.18, 1.19, 1.28	Vlcek and Cheung 1986
Ponderosa pine crowns	2.27	Zeide 1990
Hemlock crowns	2.81	Zeide 1990
Cypress patch (Florida)	1.6	Mandelbrot 1983
Broadleaf patch (Florida)	1.0	Mandelbrot 1983
Deciduous patch, below 60 ha	1.15	Krummel 1986
Deciduous patch, above 60 ha	1.4	Krummel 1986
Vegetational habitats	1.3-1.8	Morse *et al.* 1985, May 1988
Fire scars, Minnesota	1.47-1.64	Lorimer, unpublished
Fire scars, Wyoming	1.46-1.55	Lorimer, unpublished
Soil properties	1.5-2.0	Burrough 1985
Landforms, section	1.1-1.3	Phillips 1985
Landforms	2.1-2.75	Burrough 1985
River flows	1.02-1.52	Burrough 1985
Annual precipitation	1.13-1.36	Burrough 1985
Taxonomy	1.1-2.1	Burlando 1990

Two ginkgo trees, one with notched leaves and the other without notches, had different fractal dimensions for their leaf perimeters. So, the same tree species can have leaves with variable fractal dimensions. The opposite is also possible. As in the case of the oaks shown in the diagram, two different species that differ in the size and number of lobes of their leaves can have identical fractal dimensions. Although fractal geometry can be used to study leaf shape within species, it may not be useful for differentiating species (Vlcek and Cheung 1986).

Tree Branching

Crawford and Young (1990) studied tree branching by characterizing and then classifying the kinds of branching in actual trees. Segments between nodes, the points where branches emerge, were grouped in a hierarchy of orders, and each was assigned an integer label. The authors then noted relationships between order number and average length, diameter and number of segments within the order (Crawford and Young 1990).

Data from two species of oak were analyzed with two different models: a Markov model where branch length depends on the length in the previous order, and a fractal model. In the fractal model, the function giving the relative frequency of each scale in the structure is a fractal. The fractal dimension of the distribution of scales will have a bearing on the overall appearance of the tree (Crawford and Young 1990).

Power law and exponential distributions were indistinguishable for small values of the exponent. But at higher values, the branching followed the fractal model. Because branching in the two species of oaks was different, but branching measured in the lower and upper crowns of one species was the same, Crawford

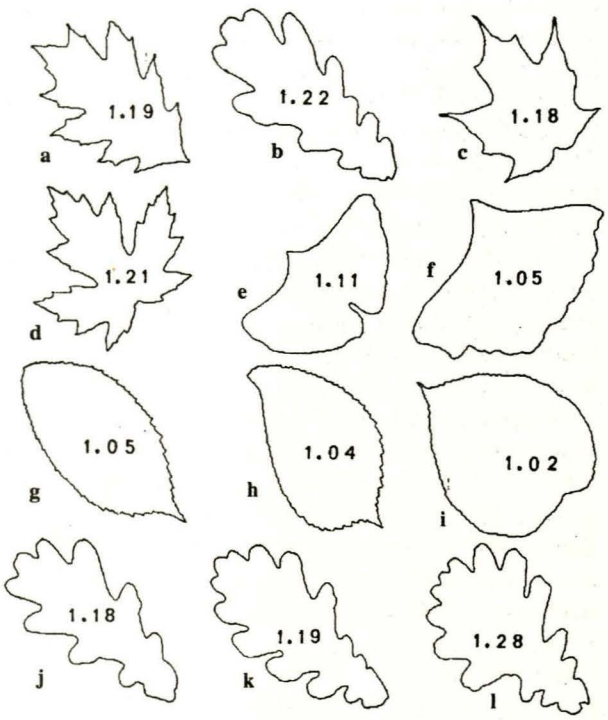

Diagram 12.—*Fractal dimensions of several tree leaves. The leaves with the most complex shapes have the largest fractal dimensions. (Reprinted with permission from: Vlcek, J.; Cheung, E. 1986. Fractal analysis of leaf shapes. Canadian Journal of Forest Research. 16: 124-127.)*

and Young conclude that branching pattern is genetically rather than environmentally determined.

The fractal model reproduced the power law behavior of mean branch length as a function of order; the branching structure showed large intraorder variability because of contributions at many scales (Crawford and Young 1990).

Tree Crowns

Another forest characteristic, arrangement of tree crowns, can be modeled as a fractal packing problem. Fill a volume of space with as many large irregular objects as possible, then fill in the remaining spaces with the next size and so forth. In the forest, packing of smaller and smaller sizes stops at the scale of the crown size of the smallest plant existing at a given height above the ground. Between the volumes of the largest and smallest crowns, there will be a difference of two or three orders of magnitude (Loehle 1983).

Before fractal geometry, foresters did not have the tools to measure complex objects like tree surfaces. Yet without adequate measures of tree surface area, it is difficult to adequately estimate tree respiration per unit area, productivity, and other properties. Like all fractal structures, tree surface measurement depends on the unit of measurement (Zeide 1990).

For tree crowns, $2 < D < 3$, volume and surface merge. There are practical problems in calculating the fractal dimension of tree crowns, however. The usual method of box-counting is difficult to apply because three-dimensional spatial data on tree crowns are difficult to obtain. Zeide (1990) developed the two-surface method. The relationship between leaf area of a tree, A, and surface area of a convex hull that encloses the crown, E, has a parameter identified as a fractal dimension of crown surface, D. This relationship is

$$A = aE^{D/2} \qquad \text{(a is a constant) (Zeide 1990).}$$

The regression of the logarithm of leaf mass (assumed to be proportional to leaf area) plotted on the logarithm of the cubic root of crown volume (representing the linear size of the crown) for 10 conifer species in the Rocky Mountains indicates self-similarity for crown surface. The slope ranged from 2.27 for intolerant ponderosa pines to 2.81 for very tolerant hemlocks. A value of three indicates uniform needle distribution, with mass proportional to volume (meaning needles tolerant to low light). A dimension of two indicates trees very intolerant of shading, all the needles appearing on the crown surface (Zeide 1990).

In a study of loblolly pine, Zeide (1991a) computed fractal dimensions of crowns on three different plantations. The fractal dimensions, 2.45, 2.63, and 2.74, increased with site quality. The fractal dimension may be sensitive to thinning (Zeide 1991a).

Tree Models

Mandelbrot (1983) modeled "trees" of different fractional dimensions and noted the effect on form (diagram 13). The internal branching angle (identical for all angles within a tree)

emerged from the restriction that no branches overlap. D increases from slightly above 1 (top left) to 2 (bottom left). The tree on the lower right, with a branching angle greater than 180 degrees, has a D less than 2 (Mandelbrot 1983).

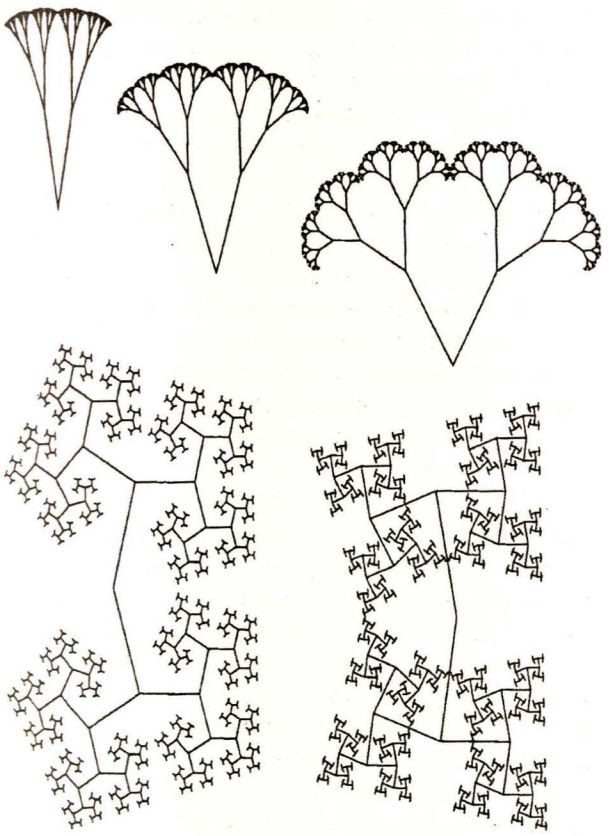

Diagram 13.—*Models of trees with non-overlapping branches. The internal branching angle changes as the fractal dimension changes. (Reprinted with permission from: Mandelbrot, B. 1983. The fractal geometry of nature. New York, NY: W.H. Freeman and Co.)*

From a topological (that is, Euclidean) point of view, the branch tips would be dimension 0 (points) and the branches would be dimension 1 (lines). The branches dominate. From a fractal point of view, the dimension of the tips is D, a fractional dimension. Each branch is still dimension 1, but the whole tree dimension is D. That is, even though the branches seem to dominate in an intuitive and topological sense, they are fractally negligible. With D greater than the Euclidean dimension, D measures the abundance of branching (Mandelbrot 1983).

When tree branches are allowed to overlap, the set of tips is no longer a dust of Euclidean dimension 0, but a curve of dimension 1 (with no change in the fractal dimension). Mandelbrot (1983) calls these curves extended fractal canopies.

Trees are both branch tips and branches, whose dimensions "clash." Varying the branch angle over a wide range while keeping branch length and D the same results in quite a variety of tree shapes (Mandelbrot 1983).

Without thin trunks, the trees in the diagram are not strictly self-similar. What of thick-stemmed tree forms in nature? Real trees illustrate the idea of a shape having many linear scales. Besides the D of the branch tips, trees have another parameter, the diameter exponent, Δ. A self-similar tree with a trunk will have $D = \Delta$. Otherwise Δ is less than D (Mandelbrot 1983).

Da Vinci claimed that "All the branches of a tree at every stage of its height when put together are equal in thickness to the trunk (below them)." To put this formally, a tree's branch diameters before and after bifurcations d, d_1, and d_2, satisfy the relation

$$d^{\Delta} = d_1^{\Delta} + d_2^{\Delta}$$

with the exponent being $\Delta = 2$ (Mandelbrot 1983).

For botanical trees, $D = 3$ and $\Delta = 2$, but the exceptions to this may be more interesting than the rule. When D and Δ are not integers, the fractal nature is more apparent. D equal to 3 would give the most surface to the sun, but there may be architectural and physiological constraints. The data for the measurement of Δ are sparse (Mandelbrot 1983).

A corollary to $D = 3$ and $\Delta = 2$ is that the branch's leaf area is proportional both to the volume of the branch's outline and to the cross-sectional area of the branch. A second corollary is that the ratio

$$\frac{(\text{tree height})^3}{(\text{trunk diameter})^2}$$

is constant for each species, and that it is equal to the ratio

$$\frac{(\text{linear scale of a branch's drainage volume})^3}{(\text{branch's diameter})^2}$$

This ratio may not vary among species (Mandelbrot 1983).

There are engineering limitations to a tree's height: a fixed percent of the critical buckling height of a uniform cylinder of the same base diameter loaded under its own weight. This requirement yields the same results as fractals with $D = 3$ and $\Delta = 2$ (Mandelbrot 1983).

Tree crown models can be constructed with simple algorithms by adding increments to a cylinder and creating forks at each increment. Plotting log frequency of branches of a given radius vs. log of that radius gives the straight line with negative slope that is characteristic of fractals. Conserving the cross-sectional area after each forking episode gives a slope of -2: the figure will fill space. If the cross-sectional area is not conserved, the slope is between -1 and -2: the figure will have a finite volume. The corresponding fractal dimension will thus range from 2 to 3. However, trunks and large branches on hardwoods do not fit the log-log relationship. Branch shedding gives long sections of clear bole (Loehle 1983).

For real trees, the length of branches in a radius class has to be measured. Loehle (1983) derived a formula for stem and root surface areas, volumes, and numbers of tips. Once basic values have been obtained, including the fractal dimension, more attributes can be estimated after measuring diameter at the base of the crown and top of the root system, plus the trunk and large branches. Forest biomass estimates can then be made with few measurements and without destructive sampling. With bark thickness as a function of branch radius, the amount of bark on a tree can be known, which is important for whole tree utilization (Loehle 1983).

A very realistic tree can be generated with a recursive algorithm in which the main tree trunk extends to smaller and smaller branches. Parameters for curves and twists of branches and the ratio of the size of the trunk to the branches are added. This tree looks like it has evolved a long way from Mandelbrot's trees. The extra information added to the simple recursive formula in the computer program is like a metaphor for increased genetic information over evolutionary time (LaBrecque 1987).

Vegetation Patches

Palmer (1988) suggests that vegetation is a prime example of a fractal, having detail at many spatial scales. From the small scale of individual plants, to the larger scale of a landscape's vegetation patches, to the yet larger scale of vegetation patterns influenced by geomorphological features, vegetation is a candidate for fractal analysis (Palmer 1988).

From the air, natural forests have boundaries similar to islands (Mandelbrot 1983). High elevation boundaries between forest and meadow are fractal (Loehle 1983). In the forest, large vegetation patches are joined by satellite patches of different sizes; species patches are irregular. Mandelbrot (1983) cites an unpublished report on patch irregularity in the Okefenokee Swamp. The perimeters of cypress patches ($D = 1.6$) were more irregular than the perimeters of broadleaf and mixed broadleaf tree patches ($D = 1$).

Size as well as species can influence the fractal dimension of a patch perimeter. When the fractal dimension of deciduous forest patches in Mississippi was computed from digitized aerial photographs, small patches had lower fractal dimension than large patches. Below 60 ha, forest patches had regular shapes: the smaller patches were generated by survey and township divisions. Above 60 ha, the "natural" patterns were more irregular (Krummel 1986).

Before the fractal concept, quantification of vegetative patterns had not been possible despite its usefulness for habitat typing, LANDSAT analysis, analysis of root disease centers, and other applications. Patchiness, an intuitively important but quantitatively vague concept, can now be quantified (Loehle 1983).

Size and Density of Organisms Related to Habitat

Sizes and densities of individuals are related to the fractal dimensions of their habitats, whether the habitat is soil (Frontier 1987) or foliage (Morse *et al.* 1985, Shorrocks *et al.* 1991).

Morse *et al.* (1985) computed fractal dimensions for plant surfaces ranging from 1.28 to 1.79, with a mean of 1.44. If a fractal surface has a linear transect with dimension 1.5, for every order of magnitude increase in measurement length, the distance from point to point increases by a factor of 10 = 3.16. There are implications for the animals that live on these plants (Morse *et al.* 1985, Wiens and Milne 1989).

Arthropods living on the surfaces of plants are like rulers, measurement sticks. Larger insects represent larger units of measurement, and smaller insects represent smaller units of measurement. On vegetation with a fractal dimension of 1.5, the area perceived by a 3-mm animal will be an order of magnitude larger than the same area perceived by an animal 30 mm long, increasing the available space for small animals (Morse *et al.* 1985).

This can be carried one step further. If metabolic rate scales as 0.75 of body weight, and if population density is proportional to the reciprocal of metabolic rate (this being related to resource utilization), then population density scales as $(L^3)^{-0.75}$ (L is body length). A 10-fold decrease in body length results in a $(10^3)^{0.75}$ = 178-fold increase in the density of individuals. So the relative numbers of animals of different body lengths living on vegetation surfaces can be predicted. Order of magnitude decreases in body length predict increases of between 178 x 3.16 = 560-fold and 178 x 10 = 1,780-fold in animal number (Morse *et al.* 1985).

Sweep net data for numbers of insects agree with the predictions (diagram 14). We may reverse the calculations to obtain fractal dimensions of vegetation, given the numbers and body lengths of collected insects (Morse *et al.* 1985). The fractal dimension of vegetation surfaces, which may differ in different locations, may be an important "ecological indicator" (Williamson and Lawton 1991).

Shorrocks *et al.* (1991) obtained similar data for insects on lichen. The fractal dimensions measured for lichen transects ranged from 1.37 to 1.78, with 1.58 the average. The body size vs. insect frequency graphs for collembola and mites collected from the lichen matched the predictions from the fractal dimension of the vegetation.

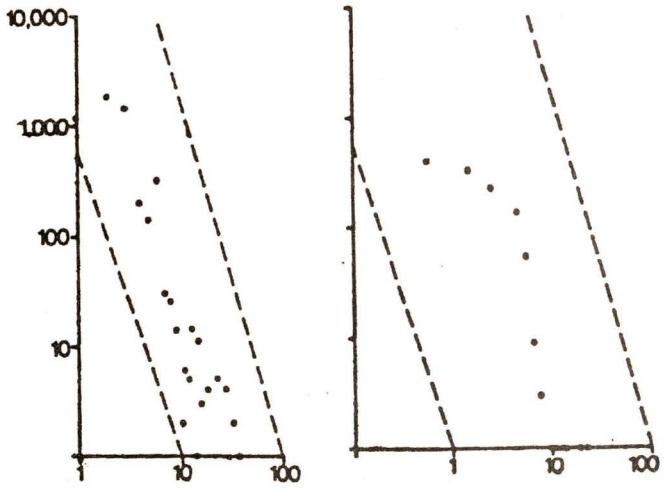

Diagram 14.—*The negative log-log relationship between numbers of arthropods and body length collected by sweep nets from understory foliage in a primary forest in Costa Rica (left) and by pyrethrum knockdown from a forest canopy (*Betula pubescens*) in England (right). Reprinted with permission from: Morris, D.R.,* et al. *1985. Fractal dimension of arthropod body lengths. Nature. 314: 731-733.*

Vegetation surfaces can change over time as well as from place to place. The fractal dimension of a tree, for example, can change by season (Williamson and Lawton 1991).

More data are needed on fractal dimensions of habitats and the body sizes of animals that live in those habitats. Lawton (1986) also suggested the formulation of a general theory of plant architecture that relates the size, variety, and fractal nature of plant structures.

Species Diversity

Possible applications of fractals in community ecology include analyses of geographical distributions of species and resource partitioning among species. Williamson and Lawton (1991) note that when a species distribution is mapped at different scales, pairs of maps appear self-similar. If so, a pattern at one scale can be used to predict patterns at larger or finer scales. The authors further speculate that species-area distributions and the allocation of environmental resources between species are fractal processes.

Lawton (1986) addresses the question of why there are so many more little species than big species in animal communities. For plant-feeding insects, the explanation may depend in part on the observation that plant surfaces are fractals. Because plant surface is a fractal, more absolute habitat surface area is available to small insects than to large insects. Further, small insects require fewer resources to survive than large insects. The greater habitat area and smaller resource requirement combine to create a larger habitat carrying capacity for small insects. Observations of insect communities support this prediction: the number of small insects increases exponentially with decreasing body size. Although this argument may explain the large number of small individuals, the argument accounts for less than half of the observed increase in the number of small relative to large species. The unascribed increase can be due to increasing resource variety at smaller habitat scales.

Another way to show distribution of individuals among species is a rank-frequency distribution. Species are ranked by frequency, then each species is given a point on the graph, with rank on the abscissa and frequency on the ordinate, after log transformation. The shape—convex, concave, or stepped—gives information about species distribution.

Rank-frequency distributions may have a fractal origin (Frontier 1987). Such distributions can be described with a curve predicting the probability of species occurrence Pr(Sr) as a function of its rank r,

$$Pr(Sr) = P_0(r + b)^g,$$

where P_0, b, and g are parameters to be estimated. The parameters define the shape of the curve: g is a negatively sloped asymptote, and b is the direction of approach. Frontier (1987) proposes ecological interpretations of the parameters: b is linked to the diversity of the environment (i.e., the number of conditions required for species occurrence), and 1/g is linked to the predictability of the community (i.e., the probability of species occurrence when the conditions that it requires are present). Finally, Frontier (1987) states, without proof, that 1/g is the fractal dimension of the distribution of individuals among species.

Fractal taxonomy gives us more information about the organization of species diversity. Many taxa have only one or two subtaxa; few taxa have many subtaxa, a pattern shown by plants and animals at several taxonomic levels. Log-log plots of frequencies of genera against numbers of species fit straight lines with negative slope, a relationship common to fractals (Burlando 1990, Minelli *et al.* 1991) (diagram 15).

The equation for these plots is

$$N = KS^{-D}$$

where N is the frequency of the genera, S the number of species per genus, K a constant, and $-D$ the exponent corresponding to the log-log regression line slope b. The absolute value of D is the fractal dimension (Burlando 1990).

Fractal dimensions computed from 44 taxonomic checklists and catalogs ranged from 1.1 to 2.14. Some examples: 70,029 species of South American Coleoptera in 6,439 genera had fractal dimension of 1.46; 4,258 species of world mammals in 1,013 genera had fractal dimension of 1.66. Similar values occur for the same groups from different geographical areas, collected by different taxonomists. Arthropods and higher plants show D estimates near 1.5. Vertebrates had the highest values, except for amphibians and reptiles, which had the lowest. The value for families within orders was similar to the value for species within genera. For example, 777 families of world plants in 145 orders had a fractal dimension of 1.21 (Burlando 1990).

Why should taxonomic diversity be fractal? In an analogy similar to the concept of fractal tiling, taxa with different numbers of subtaxa are like different-sized grains of sand. Large grains packed together leave empty spaces among themselves where smaller grains can fit. The smaller grains leave spaces taken by even smaller grains, and so on, until the smallest grains (monotypic taxa) fill the smallest spaces (Burlando 1990).

Animal Movement

In a fractal framework, surface distances depend on the unit of measurement, so a fractal approach may be useful in the study of animal mobility (Weiss and Murphy 1988).

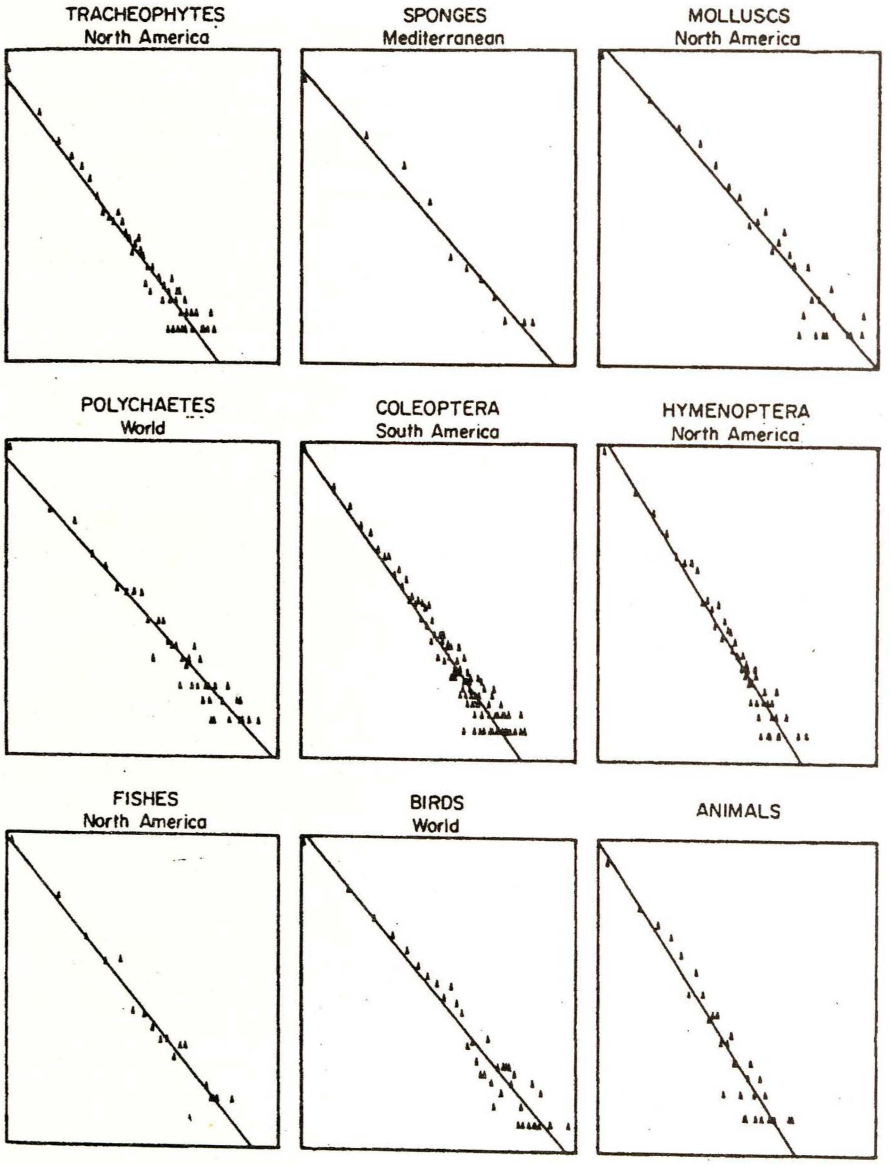

Diagram 15.—*Fractal taxonomy: log-log plots of frequencies of genera with numbers of species. The graph on the lower right shows the frequencies of orders with numbers of families. A straight line with a negative slope is common in fractal relationships. (Reprinted with permission from: Burlando, B. 1990. The fractal dimensions of taxonomic systems. Journal of Theoretical Biology. 146: 99-114.)*

Caterpillar dispersal is a critical mechanism in the population dynamics of many Lepidoptera. Large larvae can move farther; first-instar larvae have to be close to food. The sooner caterpillars find food, the less the mortality and the faster the development time (Weiss and Murphy 1988).

Caterpillar paths can be computed for given fractal dimensions, with the length of the caterpillar as the step-length. Completely smooth surfaces would have D = 1. Diagram 16 shows actual distances traveled by caterpillars of different sizes over fractal paths of 1.1 to 1.5, to cover a linear meter. Even at D = 1.1, representing only slight roughness, 1-mm caterpillars travel 2 m to cover a linear meter. Caterpillars 3 cm long travel 1.42 m. As D increases, distance traveled increases (Weiss and Murphy 1988).

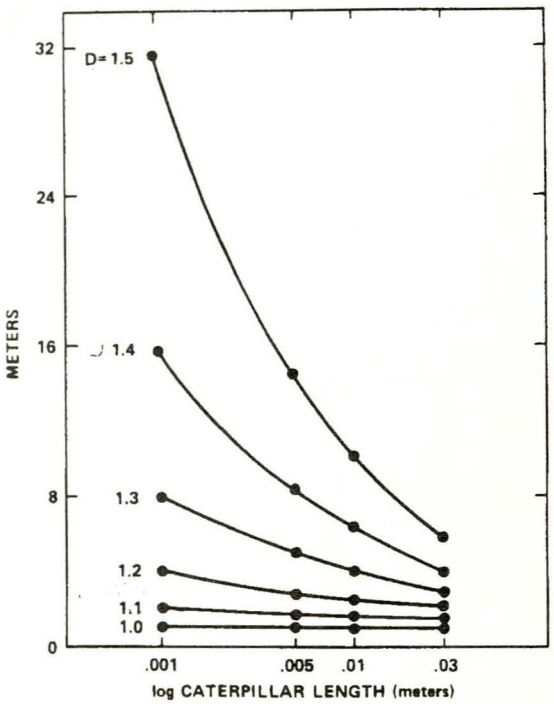

Diagram 16.—*Over rough surfaces, caterpillars of different lengths travel different distances. (Reprinted with permission from: Weiss, S.B.; Murphy, D.D. 1988. Fractal geometry and caterpillar dispersal. Functional Ecology. 2: 116-118.)*

Dispersal distances can be expressed in terms of body length. A caterpillar 1 cm long must travel 251 body lengths to move 1 m along a surface with a fractal dimension of 1.2. The same insect would have to crawl 1,000 body lengths to move 1 m along a surface with a fractal dimension of 1.5. Therefore, the distance an animal travels depends both upon its size and the texture of the substrate (Weiss and Murphy 1988).

For larger animals, home ranges are usually represented by bell curves or polygons, but they are actually more fragmented and irregular. Nor is home range merely a bounded space within which an animal may be found. Some areas are used extensively, some areas are never used. Animal ranges have fractal characteristics (Loehle 1990).

A third example of fractal ramifications of animal movement is Frontier's (1987) discussion of complex and hierarchical predator behavior. When predators encounter prey, broad hunting behavior is replaced by local hunting behavior—straight scanning movements become Brownian-type movements. These search patterns are probably fractal, correlated with the fractal pattern of the prey.

Forest Disturbance

Applying fractal concepts to spatial patterns enables not only more precise descriptions, but also hypotheses about causes of pattern. An example is forest disturbance by wind. Wind turbulence is self-similar; small eddies whirl within larger eddies. Hence, wind may cause damage at various spatial scales: it can break twigs or down a whole forest. Loehle (1983) postulated that wind damage may be self-similar.

The effect of wind turbulence on breakage in the forest can be studied by quantifying canopy roughness. Patterns of wind damage in different forests can be explained in terms of interaction between a fractal, self-similar process (wind turbulence) and a fractal surface (the forest canopy) (Loehle 1983).

Besides wind, agents such as insects, disease, and fire create openings in the forest. The fractal nature of these openings, the irregularity of the boundary between damaged and undamaged areas, may contribute to vegetational patchiness. However, not much data are available on the fractal nature of these kinds of forest disturbance. Lorimer (unpublished) computed the fractal dimension of the perimeters of spatial fire patterns in Minnesota (Heinselman 1973) and Wyoming (Romme 1982). D ranged from 1.46 to 1.64.

MacKay and Jan (1984) modeled forest fire dynamics with the techniques of percolation theory. In their protocol, a burning tree in a densely packed lattice was able to ignite its nearest neighbors according to an assigned probability. The fire clusters of burned vs. non-burned trees resulting from model parameters had fractal dimensionality of 1.75. At the critical point, called the percolation threshold, meaning the point of fire propagation, there was statistical self-similarity. A collection of trees can therefore be modeled like a ferromagnet. Instead of up-spin and down-spin, trees are either burned or not burned (Schroeder 1991).

Modeling Soils with Sierpinski Carpets

Before the fractal concept was developed, it was difficult to relate particle-size distribution in soils to water retention data. Much information is available on particle size distribution, but measures of water retention are difficult to obtain. Tyler and Wheatcraft (1990a) model the relationship with a classic fractal form, the Sierpinski carpet.

Using different Sierpinski carpets as models of different soils (diagram 17), the authors calculate water retention data from the models (diagram 18). The fractal dimensions of 1.46, 1.89, 1.96, and 1.99 measure the ratio between characteristic pore size and pore area. The fractal dimensions are measures of soil texture. The nearer the value is to 1, the more the soil is dominated by large pores: the carpet will be sparse at large scales of measurement. The diagram shows how the fractal dimension affects the shape of the curve for water retention. Near a D of 1.5, the water retention curve is like the curve for a coarse sand. Near a fractal dimension of 2, the curve is like the curve for a clay soil (Tyler and Wheatcraft 1990a).

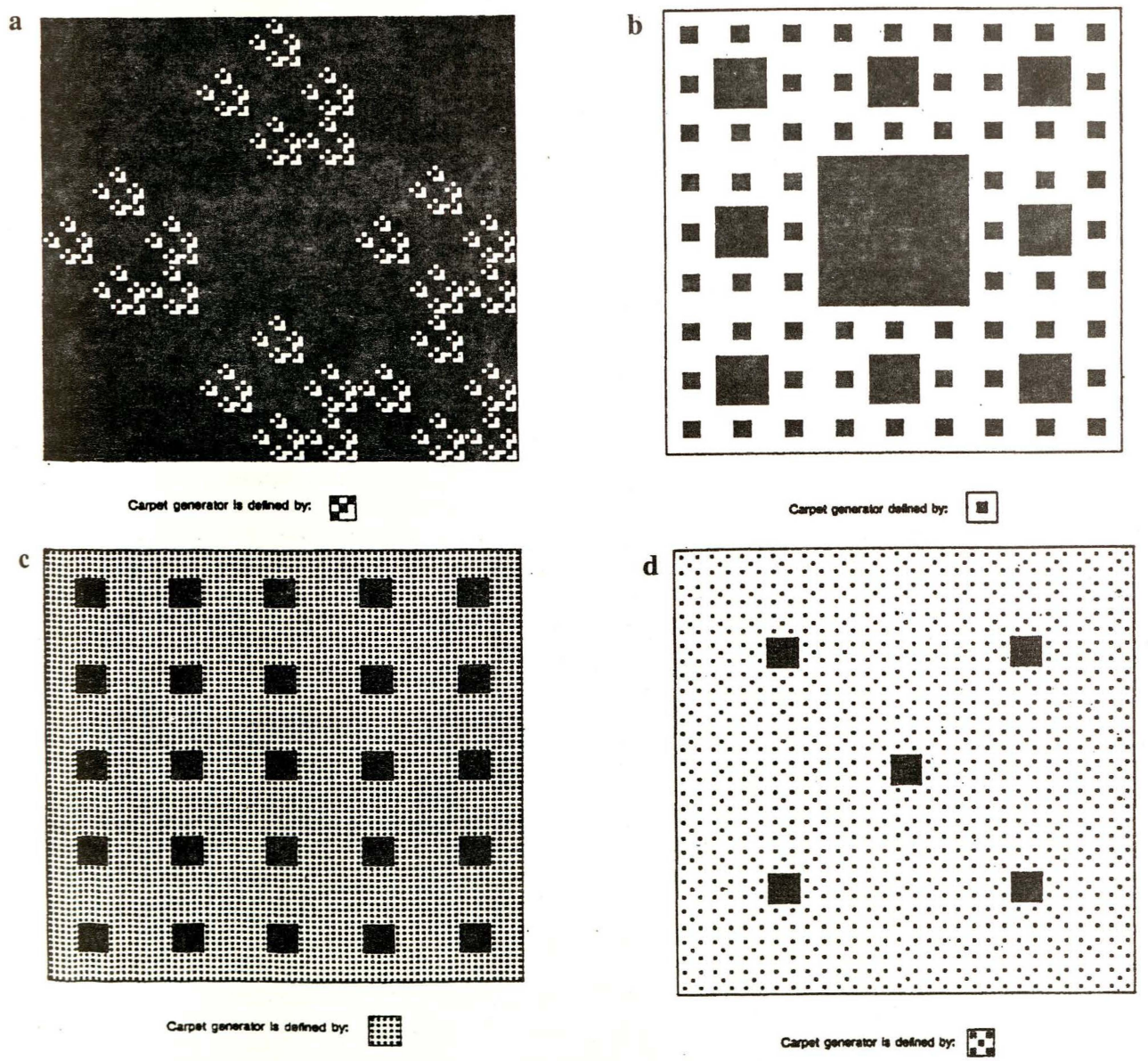

Diagram 17.—*Sierpinski carpets modeling soils of different fractional dimensions. The dimension of carpet (a) is 1.46, (b) is 1.89, (c) is 1.96, and (d) is 1.99. (Reprinted with permission from: Tyler, S.W.; Wheatcraft, S.W. 1990a. Fractal processes in soil water retention. Water Resources Research. 26: 1047-1054.)*

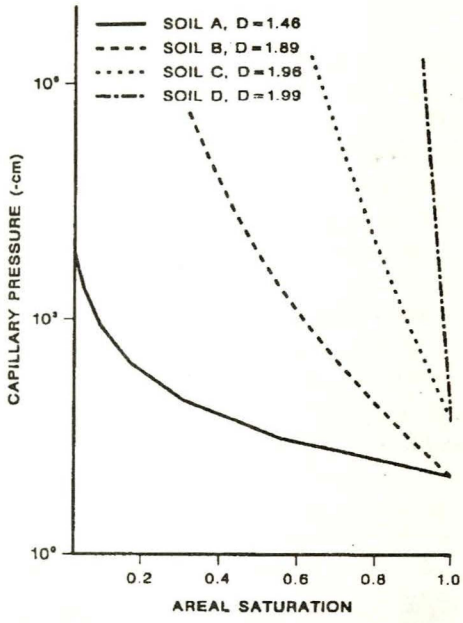

Diagram 18.—*Relationship between water retention and fractal dimension in soils. Soils with lower fractal dimensions are dominated by large pores. Soil A returns water like coarse sand. Soil B returns water like clay. (Reprinted with permission from: Tyler, S.W.; Wheatcraft, S.W. 1990a. Fractal processes in soil water retnetion. Water Resources Research. 26: 1047-1054.)*

The fractal geometry of soil structure can be carried further. It can be compared with other soil properties like surface charge and aggregate stability to examine processes of soil creation and destruction (Bartoli *et al.* 1991). The spatial organization of soil can change over very short time scales because of weathering, root growth, and soil management practices like cultivation. Fractals may make it possible to quantify soil heterogeneity and to relate structure to specific soil processes (Young and Crawford 1991).

Spatial Heterogeneity and Sampling

In this section, we will investigate the applicability of fractal techniques to issues regarding the sampling of heterogeneity. Most of the examples are from soil science and geological research.

What is homogeneity? It can mean lacking variation, well mixed, continuous, or consistent. It can also mean "remaining similar upon subdivision." Consider two forests of 20 ha each, each with 20 species of trees. In the first forest, the species have the same mix in each hectare. The forest is homogeneous. In the second forest, each species occurs only within its own 1-ha unit. The second forest is heterogeneous. But this concept of homogeneity is linked to scale. In the first forest, heterogeneity emerges on a scale slightly larger than individual trees; individual trees of one species will be neighbors of other species. In the second forest, at the scale of several individual trees, only one species is likely; at that scale the second forest is homogeneous (Palmer 1988).

The systems we study can "look" homogeneous or heterogeneous, depending on the "averaging window" or plot size (diagram 19). Fractal geometry gives us a new way to view heterogeneity. In a fractal case, heterogeneity looks the same over a range of scales (Tyler and Wheatcraft 1990b) (diagram 20). Traditional scaling techniques cannot be applied to this kind of heterogeneity.

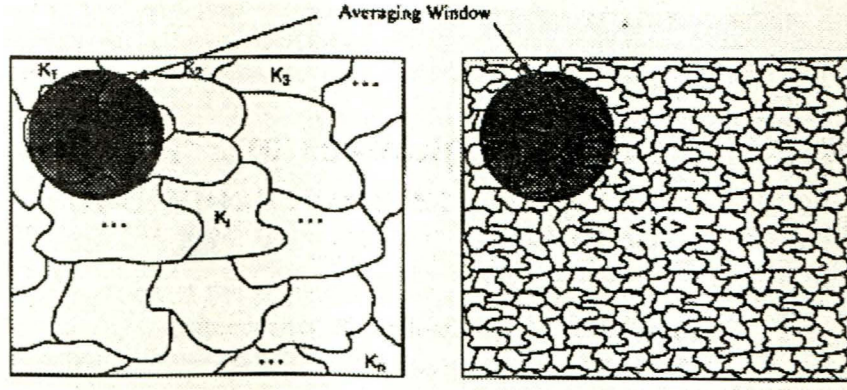

Diagram 19.—*Heterogeneity can depend on scale. At high magnification (laboratory scale), the sample on the left is homogeneous. At low magnification (field scale), the sample on the right is heterogeneous. (Reprinted with permission from: Tyler, S.W.; Wheatcraft, S.W. 1990b. The consequences of fractal scaling in heterogeneous soils and porous media. In: Hillel, D.; Elrick, D., eds. Scaling in soil physics: principles and applications, proceedings of a symposium; 1989 October 18; Las Vegas, NV. Spec. Publ. 25. Madison, WI: Soil Science Society of America: 109-122.)*

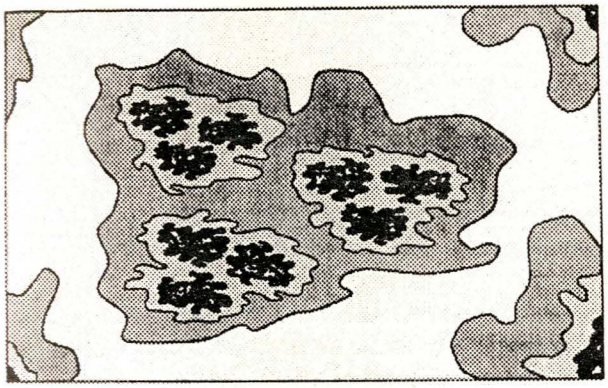

Diagram 20.—*Self-similar heterogeneity. The heterogeneity will look the same regardless of the sampling scale. (Reprinted with permission from: Tyler, S.W.; Wheatcraft, S.W. 1990b. The consequences of fractal scaling in heterogeneous soils and porous media. In: Hillel, D.; Elrick, D., eds. Scaling in soil physics: principles and applications, proceedings of a symposium; 1989 October 18; Las Vegas, NV. Spec. Publ. 25. Madison, WI: Soil Science Society of America: 109-122.)*

All vegetation surveys are subject to spatial dependence. Two nearby quadrats will have more similarity than two separated quadrats. This presents a problem for statistical analysis because an important underlying assumption, that replicates are independent, is violated. Fractal geometry may suggest alternative sampling schemes (Palmer 1988).

Fractals and geostatistics combine to describe the complexity of the spatial variation of soil. Soil-forming factors (interaction of parent materials, climate, hydrology, relief, biological acitivity) operate over different spatial scales, and within each factor may be many scales of interaction (Burrough 1983a).

When *D*'s of soil (greater than 1.5) are compared to *D*'s of other environmental variables (less than 1.5), soils appear to be "noisier." This implies that, for other environmental variables, increments along the sample series tend to be positively correlated, and that, for soils, increments along the sampled series are negatively correlated (Burrough 1983a).

Large *D* values in soils may be due to small-scale variations caused by rock weathering, biological action, erosion, and other factors. When large-scale effects dominate, *D* values are smaller; variation is less erratic (Burrough 1983a).

For soils, representative area volume (REV) is a sample size below which a sample will show wide variation as a function of its location. Above the critical size, the variable of interest will be constant. The REV approach sees soils as building blocks of homogeneous material above some critical scale (Tyler and Wheatcraft 1990a).

A good soil survey recognizes scale of change, abruptness of change, and degree of correlation among soil properties and properties of the landscape. Previously, surveyors brought intuition to this job, rather than quantitative tools (Webster 1985).

At the landscape level, *D* values can also range widely. The fractal dimension is a useful indicator of autocorrelation over many scales. Some natural phenomena do display statistical self-similarity over many spatial scales; but other natural phenomena are structured, that is, they have levels of variability clustered at particular scales. However, they are not excluded from the fractal concept because of this. Zones of distinct dimensions could be connected by transition zones. So, *D* values can be used to sort out scales of variation linked to particular natural processes. Identifying these scales would have great practical value. Sampling could then be tailored to a particular scale range, improving efficiency, saving money, and improving interpolation (Burrough 1981).

For sampling, the fractal dimension *D* is useful for showing how a given sampling design could resolve the variations present with the appropriate scales. Small *D* means that the samples found the change in the variable to be smooth, not abrupt. Large *D* means the variation is irregular and uncorrelated. A large *D* after the resolution of the survey has been adjusted, means important, short-range sources of variation. A changing *D* over closely related scales would require sample spacing with the smallest value of *D*. That kind of sample spacing would resolve long-range variations without the confusion of unresolved short-range effects (Burrough 1983a).

As applied to soils, for example, interpolation to non-sampled sites may be risky because *D* values are usually large (Burrough 1983a). Burrough (1983a) suggested finding the major scales of variation present in a landscape before beginning studies at any scale.

Fractal techniques can be combined with geostatistics to fine-tune sampling in heterogeneous environments. Commercial geologists developed geostatistics to predict the amount of metal ore in unsampled locations (Palmer 1988). Geostatistical methods use spatial autocorrelation, the theory of regionalized variables (Meentemeyer and Box 1987).

The semivariogram summarizes the variance in a dependent variable as a function of scale. If the variable changes as a linear function of distance, the semivariogram will be a parabola. The slope of the double logarithmic plot of a parabola is 2, corresponding to a fractal dimension of 1. If the values for the variable in two near samples are as different as they are in two distant samples, the slope of the semivariogram will be zero, corresponding to a fractal dimension of 2. So the fractal dimension indexes the variable's spatial dependence (Palmer 1988).

Block techniques for ecological pattern analysis are related to the semivariogram. In block analysis, variance is plotted as a function of doubling quadrat or block area. Fractal dimensions can be calculated from these diagrams. Block analysis has few replicates at large sizes, and only few scales are considered. Semivariograms are better because they plot variance as a continuous function of scale. Calculation is computer intensive because each quadrat is compared to each other quadrat (Palmer 1988).

Many semivariograms are available in the literature, so *D*'s can be calculated relative to the sampling interval (Burrough 1981). Shoreline erosion is an example of small-scale influences vs. large-scale influences in a natural system. The shoreline erosion can be studied with the fractal dimension and the variogram used together (see box).

D and Shoreline Erosion

An example from geomorphology illustrates how the semivariogram and fractal dimension together can give information about natural phenomena. Controls over relationships in geomorphology vary with spatial scale. To determine the structure of a process, geomorphologists find the scales of variation and then identify controls operating at those scales (Phillips 1986).

Shoreline erosion is an example of a complex process operating at many scales. One study (Phillips 1986) identified 29 variables involved in shore erosion of the Delaware Bay. The variables represented several scales: kilometers, meters, and phi units (to measure sand grains). Some of these variables were tidal range, maximum elevation above mean low water, shoreline width, and mean sand grain size.

Wave attack was a predominant variable acting over many scales, as shown by the semivariogram (diagram 21). In a semivariogram, variance is plotted against scale. The fractal dimension can be computed from this double-log plot and in this case is 1.91. A fractal dimension of nearly 2 means that the pattern of erosion is very complex and irregular. It is a statistically random pattern indicating negative correlation between adjacent sites, a pattern resulting from short-range, or local controls. Any long-range trends that may exist are obscured by local effects (Phillips 1986).

The semivariogram and fractal dimension show that the distribution of erosion rates is highly complex and variable; that wide variation occurs over short distances; and that if any long-range trends exist, they have little influence. For shoreline erosion, the important scale of variation is the local scale (100 m) (Phillips 1986).

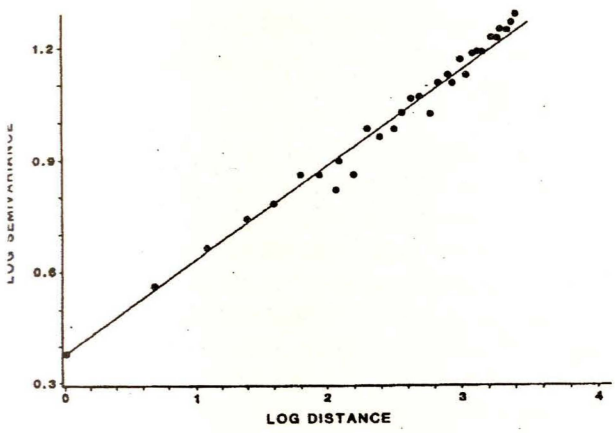

Diagram 21.—*Semivariogram of wave attack as an erosion variable acting over many scales. In a semivariogram, variance is plotted against scale. The fractal dimension (1.91) indicates a complex pattern of erosion. (Reprinted with permission from: Phillips, J.D. 1986. Spatial analysis of shoreline erosion, Delaware Bay, New Jersey. Annals of the Association of American Geographers. 76: 50-62.*

In the forest context, gradient analysis is part of a method that includes ordination and classification of vegetation. An implicit assumption is continuous spatial variation. However, there may be significant deviation from the trend at one sampling site; or a sample from a site may not represent the site's heterogeneity. A measure of the spatial heterogeneity of a variable would be useful (Phillips 1985).

The fractal dimension expresses the complexity of the data, and can be used for checking variability along a gradient. Is variability regular enough so that spatial position can be substituted for position along an environmental gradient? Does the measured gradient show deviations from the regular trend that might affect ecological interpretation (Phillips 1985)?

A linear series approaching $D = 2$ is too complex for most gradient analysis (Phillips 1985). The semivariance function helps in this context by giving the sampling density needed to represent the environmental variable (McBratney and Webster 1983). Highly complex environmental gradients may be common in nature. Geostatistics and fractal dimensions are the quantitative tools for finding and analyzing these gradients (Phillips 1985).

The techniques of fractal analysis can be applied to environmental data to quantify the scales over which one can extrapolate from the sampling site to the larger area (Krummel 1986).

Data Storage and Compression

For large data sets, like geographic information systems, a fractal approach can make data storage more efficient (Goodchild and Mark 1987).

An iterated function system is a data-compression method (Barnsley 1989, Peitgen *et al.* 1992). Iterated function systems consist of several sets of equations that rotate, translate, and scale the data according to specified probability rules (Stevens 1989). These systems are also called contraction mappings—mappings that bring points closer together. A single contraction mapping iterated on a bounded set will bring the set to a single point. But two or more contraction mappings will become very complex. The attractor is usually a fractal, and data from the contraction process help to determine the fractal dimension (Cipra 1989).

Iterated function systems have been applied to data compression of digitized pictures. Fractals from contraction mappings can be described by the small set of numbers that make up the matrices of the mappings and the time that each mapping is applied, so any fractal can be reduced to small data sets. Data-compression ratios range from 10:1 to 10,000:1. The representation can be exact or approximate (Cipra 1989).

8. FRACTAL APPLICATIONS IN MANAGING FOREST LANDSCAPES

In the previous section, we described current applications of fractal geometry in many areas of forestry research. To close this review, we focus on the potential of fractal geometry in resource management.

Maintenance of biological diversity has emerged as a significant issue in the management of federal lands. These lands encompass one-third of the Nation; and they have the potential to support the nationwide diversity of species and ecosystems while providing forest-based commodities and services. Fractal geometry may improve the management of old-growth areas and increase the understanding of species-habitat interactions.

The maintenance of old-growth forests is one goal of ecosystem management (Crow 1990). Old-growth forests have unique structural and compositional features that provide critical habitat for associated plant and animal species, and they are rare. Because of their longevity, old-growth forests may hold the key to developing practices for sustainable forestry in managed portions of the landscape.

Although old-growth, presettlement ecosystems are often the model for landscape management (Noss 1983, Noss and Harris 1986), large areas of presettlement vegetation do not exist in most landscapes. An alternative involves constructing a network of old-growth patches connected with corridors. Each old-growth patch is surrounded by a buffer zone in which human-related activities are restricted. The best management for old-growth patches and buffer zones, conserving the full complement of compositional, structural, and functional features, may involve a fractal description of heterogeneous organization.

Among the many methods of describing landscape structure (see Turner (1989) for review), fractal geometry provides several techniques for measuring the heterogeneity of vegetation. All the methods represent exponential changes in measured quantities (e.g., perimeter length, area, population density) with changes in scale. Fractals capture much of the intuitive character of natural heterogeneity, and they are one of the best tools for conducting analyses at multiple scales (Milne 1992).

Old-growth forests have been described as mosaics of groups of trees of various age classes and species (Bonnicksen and Stone 1982), and fractal geometry has been used to quantify the size-shape relationship of vegetation patches (Krummel *et al.* 1987, O'Neill *et al.* 1988) and the shapes of boundaries between patches (Loehle 1983). Consequently, fractal descriptions of the juxtaposition of vegetation patches may be used as a basis for reconstructing old-growth forests. For example, with measures of fractal dimension as a basis, locating and designing timber harvests in multiple-use areas can lessen the ecological impact in relation to old-growth areas (e.g., Franklin and Forman 1987). Fractal geometry may help explain the effect of landscape pattern on the spread of disturbances such as fire or insect damage (see, for example, Turner *et al.* (1989)). Results can be used as a basis for mimicking natural disturbance.

In addition to old-growth management, a second area of concern is the maintenance of species diversity. Research focuses on the relationship between population persistence and the amount, quality, and juxtaposition of habitat. Studies of the demography of an animal species often focus on the effects of the spatial pattern of vegetation on distribution, movement, and persistence. There is a great deal of literature on population response to patchy environments (see Wiens (1976) and Turner (1989) for reviews). Habitat suitability models relate vegetation attributes to the presence of the animal of interest (Verner *et al.* 1986). Studies of species-habitat interactions usually measure vegetation at one scale. Because the scale at which organisms interact with their environment is not known, a focus on one scale may yield equivocal results.

Fractal geometry may improve the prediction of species-habitat interactions because of its ability to unify patterns of habitat heterogeneity at a variety of scales. For example, Wiens and Milne (1989) found that beetle movement through a grassland is related to the fractal dimension of grass cover. The inference (although not explicitly shown) is that beetle movement is not related to cover at any one level of resolution. Movement could be predicted only by integrating cover across scale using the fractal dimension.

The fractal dimensions of vegetation may be used as explanatory variables for a wide range of questions about individual behavior within a species (Wiens and Milne 1989). Does the degree of fragmentation at a large scale affect organism movement on a smaller scale? Do movements change depending on the objective (e.g., search for cover, food, or mate)? Do population density and stability depend on the fractal dimension of the vegetation?

There are also interesting questions about comparative behavior across species (Wiens 1989). Do organisms that operate at different scales within the same vegetation type (e.g. beetles, jack rabbits, antelope) respond in the same way to changes in vegetation structure through habitat fragmentation? Can differences be explained by life history strategies, body sizes, physiology, or social organization? Generalities about the relationship between species persistence and vegetation can be used as a basis for better management plans.

While Wiens and Milne (1989) observe and relate a beetle's movement to the fractal dimension of its habitat, the causal mechanisms underlying the relationship are not known. It will take great effort to conduct similar studies with larger organisms or multiple species. Instead, simulation studies can generate hypotheses about the effects of spatial patterns of resources on the demography of multiple, interacting species. Simulation studies can also evaluate alternative vegetation patterns, with coexistence of multiple species as the criterion. Palmer (1992) simulates the effects of a range of spatial patterns on the persistence of species with different fitness functions. He assumes that species fitness (i.e., competitive ability and fecundity) is related to an aggregate environmental variable in a deterministic way, and that the landscape is subdivided into patches each with a particular value of the aggregate variable. The variable values are spatially arranged so that the landscape has a certain fractal dimension, which represents the degree of spatial correlation. The advantage of using fractal landscapes is that the environmental variable is continuously distributed in a statistically self-similar way rather than being either homogeneous or random. The fractal distributions of the resource look much more like real landscapes. Palmer suggests that the persistence patterns result from basic ecological principles such as habitat area, mass effect, ecological equivalency, and global biological constraints.

Milne (1992) provides an innovative description of resource availability for multiple species at different scales using fractal geometry. For each cell in a landscape, he quantifies resource availability in surrounding cells for a given size of home range. The fractal dimension of the landscape is estimated as a function of the location and density of resource availability. The fractal dimension describes the spatial variability of the resource. Next, Milne repeats the measurement for different sizes of home range. Finally, he computes elevational maps of the resource densities for different home range sizes. These provide multi-scale views of areas of interior habitat, gaps, and corridors, views crucial to resource managers interested in the effects of alternative management policies on a range of species.

Suppose that species persistence is enhanced when vegetation has a certain fractal dimension. How is a desired vegetation structure created from a given starting point? Although no one has looked at this problem directly, elements have been addressed. Palmer (1992) simulates fractal landscapes using the midpoint displacement method with successive random additions (Feder 1988, Saupe 1988). The method was originally developed to generate landscapes with elevation varying as a function of location. Instead of elevation, a general environmental variable is defined for the habitat value of a microsite (Palmer 1992). The variable depends on its two-dimensional location so that the fractal dimension of the landscape has a particular value. This is a landscape target for vegetation management. Methods are needed for other quantities (e.g., perimeter length or area) used to estimate fractal dimensions. An even more difficult problem is to determine actions that maintain a desired fractal dimension over time, given growth and disturbance functions.

To summarize this chapter, fractal geometry has been successfully applied to the description of heterogeneous landscapes. Along with other measures of landscape structure, fractal geometry will help in reconstructing old-growth ecosystems and in understanding and predicting species behavior and persistence. Work to date has shown that fractal geometry is useful

for describing the spatial pattern of resources (Palmer 1992, Milne 1992). Further research is needed to understand the ecological basis of these patterns, and their place in ecosystem management.

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Lorimer, Nancy D.; Haight, Robert G.; Leary, Rolfe A.
1994. **The fractal forest: fractal geometry and applications in forest science**. Gen. Tech. Rep. NC-170. St. Paul, MN: U.S. Department of Agriculture, Forest Service, North Central Forest Experiment Station. 43 p.

Fractal geometry is a tool for describing and analyzing irregularity. Because most of what we measure in the forest is discontinuous, jagged, and fragmented, fractal geometry has potential for improving the precision of measurement and description. This study reviews the literature on fractal geometry and its applications to forest measurements.

KEY WORDS: Fractal geometry, forest mensuration, wildlife habitat description, vegetation mapping, microclimatology.

Our job at the North Central Forest Experiment Station is discovering and creating new knowledge and technology in the field of natural resources and conveying this information to the people who can use it. As a new generation of forests emerges in our region, managers are confronted with two unique challenges: (1) Dealing with the great diversity in composition, quality, and ownership of the forests, and (2) Reconciling the conflicting demands of the people who use them. Helping the forest manager meet these challenges while protecting the environment is what research at North Central is all about.